



The logic of “improper cross”

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1. The problem

Cross-examination of witnesses is not an opportunity to explore the beliefs of those on the stand on even relevant matters. Rather, it is an opportunity to impeach evidence given by the witness during direct examination. That is possible in two ways: Questioning the credibility of the witness directly – for example by bringing up a perjury conviction or by asking what favors he received in expectation of his future testimony – or questioning the witness’s credibility indirectly, by trying to disassemble the story he has woven on examination-in-chief.

We are concerned here with formalizing this last process. Like the question of the whole truth that we raised earlier¹, the questioner is allowed considerable latitude, but that latitude *is* bounded by the examination-in-chief. Also like that question, any formalization will have to use a logic which models questions and answers, not just propositions.

Such a logic is called an *erotetic logic*, and we review the basics of the erotetic logic we will be using² briefly. A yes-or-no question asking whether or not Q is represented by $Q?$. The “pose of the question” is the range of possible answers disjoined, here “yes” or “no”. More complicated questions are based on the predicate logic and have the form $(?x)Qx$, meaning “For which x , is it the case that Qx ?”. The pose in this case may be $a \vee b \vee c \vee d \vee \dots$. The pose of $(?x)Qx$ may be conveniently represented by $(\exists x)Qx$, with the understanding that the existential quantifier is a disjunction over the domain of discourse. The “true answers” are “elements of the pose” conjoined, e.g., $b \ \& \ d$. In cases with a yes-or-no answer, the true answer will be a single element of the pose of the question. We can represent the answer to $Q?$ as $|Q?|$ and the answers to $(?x)Qx$ as $|(?x)Qx|$. A “restriction of the pose” is a disjunction of some of the elements of the pose. The maximum restriction is the pose itself and is what is returned in response to a question to

¹ Joseph S. Fulda, “The Logic of the Whole Truth”, *Rutgers Computer and Technology Law Journal* 15: 435–446 (1989). Our criterion is set forth in the text of the sentences preceding and following n. 3.

² See Nuel Belnap and Thomas Steel, *The Logic of Questions and Answers* (Yale University Press, 1976).

which the witness doesn't know the answer. The minimum restriction of the pose contains no elements and is the logical constant **F** and is what is returned when all possible answers (and hence their disjunction, the pose itself) are false. This occurs when the question has a presupposition that is false, that is a proposition that must be true for the question to have an answer. Thus the answer to the question "What did you eat for breakfast?" is **F** if you skipped breakfast – not any of the elements of the pose (breakfast foods). In such cases, the question $Q?$ is *logically false* because P , the *presupposition* of $Q?$, is false. (We have been referring to elements of the pose as "breakfast foods" or "a" or "b & d" or "yes", but this is just abbreviated form for a full sentence rephrasing the question in statement form and adding in the constant: "The breakfast food I ate is cereal" is abbreviated by "cereal".) With a few exceptions, P must generally be established before $Q?$ can be properly asked of a witness. If it is not established, the objection, "Question assumes something not in evidence", will likely be sustained.

A witness tells "the whole truth" in answer to a question put to him if he replies with the maximal restriction of the pose consistent with his beliefs.³ A witness who restricts the pose in accordance with his beliefs in response to a question but does not do so maximally tells the truth but not the whole truth.

The question we wish to consider here, on which there has been no prior academic work, is how the body of testimony offered by one party's witnesses under direct examination bounds the questions that can be asked during cross-examination of those witnesses by the other party.

2. The solution

First, it should be understood that the body of testimony being impeached is in implicit conjunction and consists, as above, of the questions rephrased as statements with the answers filled in. Second, the testimony can be impeached by contradicting not just a proposition in the testimony space, but by contradicting *any logical consequence of any proposition in the testimony space*. This is where the difficulty inherent in the problem lies. Consider the following case:

On direct examination, a prosecution witness testifies that the defendant was at the scene of the crime when the crime took place: $S(t)$

On cross-examination, a photograph is produced showing the defendant somewhere, elsewhere, next to a clock in the background which shows the time as just when the crime occurred: $E(t)$

A rebuttal witness will testify that the clock in the picture was working well: $C(t)$

Now cross-examining the witness has not produced a direct contradiction of $S(t)$. Rather, it is an *inference*. We are suggesting, of course, that $(E(t) \ \& \ C(t)) \supset \sim S(t)$. Even though there has been no testimony that $\sim S(t)$, the line of questioning is clearly proper since it clearly impeaches the prosecution witness's testimony on

³ See Joseph S. Fulda, *supra*, n. 1.

direct. On the other hand, the objection of “improper cross” should be sustained if on direct examination nothing about the witness’s whereabouts was brought up *or is logically implied by any testimony*⁴ offered.

Generalizing, suppose there are two parties, p_1 and p_2 ,⁵ with $n(p_i)$ witnesses respectively. Then the testimony of each witness is given by $T(Wp_{ij})$ where $1 \leq i \leq 2$ and $1 \leq j \leq n(p_i)$. When impeaching the testimony of witness a of p_i – i.e., when a is being cross-examined and hence is now effectively a witness for p_i' , all of $\bigwedge_j T(Wp_{ij})$ – i.e., all of the testimony produced by p_i ’s witnesses on direct examination – as well as $T(Wp_i'a)$ for witness a ⁶ – witness a ’s prior testimony on cross, may be considered.

In the above case, the party doing the cross-examination must have presented both $E(t)$ and $C(t)$ in its case and may ask the witness $S(t)?$, the answer to which, if truthful, will contradict $S(t)$. In the case of hypothetical questions, the protasis of the hypothetical is also conjoined to the testimony space from which impeachment is launched. Hence, if the question is $(P \supset Q)?$, P is conjoined to $\bigwedge_j T(Wp_{ij}) \& T(Wp_i'a)$ and the result is checked for consistency against $|Q?|$. We do not require that the answer to a permissible question lead to an inconsistency only that it *may* lead to one. Hence, the governing rule is that the question $(?x)Qx$ ⁷ may be asked if

⁴ Note that we keep referring to “testimony” as if it were interchangeable with “evidence”. The reason for this is that although physical evidence – exhibits – such as the photograph is also used, in the Anglophone world, exhibits are turned into testimony as the witness under cross-examination will be asked, for example:

Do you recognize the man in this photograph, Exhibit A for the Defense, as the defendant?

Would you tell the Court what time the clock reads on this photograph?

Isn't that the same time you testified that the defendant was at the scene of the crime, namely t ?

Would you say that the photograph shows the defendant at the scene of the crime?

The method is to force a contradiction during cross-examination, rather than allow exhibits to speak for themselves.

⁵ In the discussion below if $i = 1$, $i' = 2$, and *vice versa*; $i' = 3 - i$.

⁶ Most judges do not enforce the rules of proper cross-examination so particularly, i.e., so long as an inconsistency is being shown by the line of questioning, they will allow it, even if it is based, in part, on the testimony of the other’s sides *other* witnesses – rather than force counsel to call such witness as his own (hostile) witness. This presents no real change to the model developed here. It just means that in place of $T(Wp_i'a)$ for a particular witness a undergoing cross-examination, we have $\bigwedge_j T(Wp_i'j)$, where $1 \leq j \leq n(p_i')$ not $n(p_i)$.

⁷ We limit ourselves to monadic logic and singly general questions *without loss of generality*. Consider, for example, the dyadic predicate x IS SUSPECTED OF CRIME y . In any given question, only one of those two variables will be queried. Thus in a rape-and-murder case, where p_i maintains that a committed the crimes and p_i' is trying to show that the crimes may have been committed by different people and that the police improperly ruled out suspects for each crime that were not also suspects for the other, the question will be “Who were the suspects for the rape?” and “Who were the suspects for the murder?” or “Which suspects were not suspects for both?” In each of these cases, the crime in question is held constant while a query is made about the identity of the suspect. Thus instead of $(?x)(?y)(S(x, y))$ we have $(?x)(S(x, r))$ and $(?x)(S(x, m))$ or $(?x)(S(x, r) \oplus S(x, m))$ and, of

and only if $(\exists x)(Qx \ \& \ \bigwedge_j T(Wp_{ij}) \ \& \ T(Wp_{i'a}) \vdash^* \mathbf{F})$ or, alternatively, $(\exists x)(Qx \vdash^* \sim(\bigwedge_j T(Wp_{ij}) \ \& \ T(Wp_{i'a})))$, where \vdash^* means can be inferred in 0 or more deductive steps, i.e., is the transitive-reflexive closure on one-step inferences.

But this is just the beginning, for although any number of deductive steps are allowed, the contradiction or inconsistency must be apparent after a single question. A masterful cross-examination, of course, asks a long series of questions, and only after the last in the line of questioning is an inconsistency apparent. Indeed, it is misguided, is it not, to speak of an allowable question? What is allowed is a *line of questioning*, the answers to which may provide the desired inconsistency. But once one of the answers is off, the line of questioning must be discontinued. A line of questioning, $(?x)Q_1(x)$, $(?x)Q_2(x)$, $(?x)Q_3(x)$, \dots is *admissible* only if $(\exists x_1)(\exists x_2)(\exists x_3) \dots (Q_1x_1 \ \& \ Q_2x_2 \ \& \ Q_3x_3 \ \& \ \dots \ \& \ \bigwedge_j T(Wp_{ij}) \ \& \ T(Wp_{i'a}) \vdash^* \mathbf{F})$ or, alternatively, $(\exists x_1)(\exists x_2)(\exists x_3) \dots (Q_1x_1 \ \& \ Q_2x_2 \ \& \ Q_3x_3 \ \& \ \dots) \vdash^* \sim(\bigwedge_j T(Wp_{ij}) \ \& \ T(Wp_{i'a}))$. A line of questioning is admissible, we said, *only if* the above condition is met, but not necessarily *if* it is met. Only those otherwise admissible lines of questioning which are not derailed by an answer inconsistent with producing the desired inconsistency are actually permitted. This is because once the line of questioning is derailed, it must be stopped or we have a so-called fishing expedition, not a proper cross.⁸

We asked a rhetorical question earlier: Isn't it lines of questioning and not questions that are proper? The answer, finally, is "No; it is not". Although lines of questioning show the relevance of the questions in the line and may be needed to persuade the Court to admit the questions, it is individual questions the answers to which are in evidence and individual questions to which objections are made. This simple insight allows us to complete our characterization of an admissible question. A question is *admissible* if it is part of an admissible line of questioning and no prior question in the line has derailed the line. Formally, $(?x)Q_k(x)$ is *admissible* if and only if $(|(?x)Q_1(x)| \ \& \ |(?x)Q_2(x)| \ \& \ |(?x)Q_3(x)| \ \& \ \dots \ |(?x)Q_{k-1}(x)| \ \& \ \bigwedge_j T(Wp_{ij}) \ \& \ T(Wp_{i'a}) \ \& \ (\exists x)Q_k(x) \ \& \ (\exists x)Q_{k+1}(x) \ \& \ \dots) \vdash^* \mathbf{F}$, or, alternatively, $(|(?x)Q_1(x)| \ \& \ |(?x)Q_2(x)| \ \& \ |(?x)Q_3(x)| \ \& \ \dots \ |(?x)Q_{k-1}(x)| \ \& \ (\exists x)Q_k(x) \ \& \ (\exists x)Q_{k+1}(x) \ \& \ \dots) \vdash^* \sim(\bigwedge_j T(Wp_{ij}) \ \& \ T(Wp_{i'a}))$ that is, if and only if the prior

course, $S(x, r)$ can be taken as just $R(x)$ and $S(x, m)$ can be taken as just $M(x)$. Now a lawyer trying to fluster an officer can certainly query on multiple variables at once: "Which suspects did you list and for which crimes?" but, mathematically, that can just as well be represented as a conjunction of two singly general monadic questions as it can be as a multiply general dyadic question.

⁸ This is a bit of an oversimplification. Lines of questioning admit of contingencies, and a good lawyer is prepared for them. There is, in general, not one right answer to each of a linear sequence of questions. Rather the "line" of questioning is more likely to be a tree with bifurcations, depending on the answers given by the witness in cross. It is nonetheless true that certain answers will stymie a line of questioning, i.e., there will be no branch in the tree that can continue from that answer to that question. Note that we are still justified in referring to this structure as a line; the key point about the structure is that it is directed and acyclic, i.e., a generalized line. (The reader may suppose that a plane is a generalized line and so it is, but it is not a generalization of "line" as we use it here: A line of questioning is made up of discrete points, has endpoints, and has direction: Mathematically, in other words, it is a type of digraph. The generalization of this type of graph is indeed a tree.)

answers in the line of questioning and the testimony space of the side doing the cross and the testimony of the witness being cross-examined and some possible answers to this and the rest of the line of questioning produce an inconsistency.

3. Automation

We now ask whether this process can be automated, whether, that is, a computer can tell of a question to which the objection “improper cross” is made whether it is admissible or not. The answer is a qualified “yes”. Qualified, because although the computer can evaluate logical formulae flawlessly, it cannot predict the human interlocutor’s future questions – neither, in fact, can a human judge. Often, after such an objection is raised, the Court will say, “I fail to see the relevance of this line of questioning”, and counsel will approach the bench and say, “I am trying to show such and such”, after which the Court will say, “Sir, you may proceed”. What both the judge and the computer must be given is the (rest of) the line of questioning, otherwise the formula cannot be evaluated, so when passing on the admissibility of $(?x)Q_k(x)$, some idea of the Q_j , $j \geq k + 1$ must be there so the Court (or the computer) can “see where all this is leading”.

Such automation with interaction would be best embedded in a dialogical – dialogue logic – system such as that presented in detail by Hage et al.⁹ In other words, one party would present evidence during cross-examination as a move, then the other party could question the admissibility of this evidence as a countermove. A call would be made to an inference engine to see whether the evidence presented contradicts the reflexive-transitive closure (under \vdash) of the testimony space – that is, the conjunction of all testimony offered under direct examination and prior testimony by this witness under cross. If the reflexive-transitive closure of this space shows no inconsistency with the testimony proffered, a further call is made, this time to a referee. Without the other party being able to observe, the referee asks the questioning party for the series of questions that the cross-examiner seeks to ask *if* he gets the “right” answers – answers that one after another will eventually lead to an inconsistency – to prior questions. Then applying the formulae developed above, the referee (probably with the aid of the inference engine we spoke about earlier) makes the decision. It is important to note that the referee, and not an inference engine, must make this judgment since there are many cases where an inconsistency is implicit and depends on common-sense knowledge (for example, that if the defendant is somewhere, he is not elsewhere, to refer to our prior example) that will not be in the testimony space, its reflexive-transitive closure, the testimony space augmented by the proposed questions and the “right” answers, or the reflexive-transitive closure of the augmented testimony space.

⁹ Jaap Hage, Ronald Leenes, Arno Lodder, “Hard Cases: A Procedural Approach”, *Artificial Intelligence and Law* 2: 113–167 (1993–1994), which I favorably reviewed in *Computing Reviews* 38: 524 (1997).

