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Approximate syllogisms – on the logic of everyday life

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Abstract. Since Aristotle it is recognised that a valid syllogism cannot have two particular premises. However, that is not how a lay person sees it; at least as long as the premises read “many”, “most” etc, instead of a plain “some”. The lay people are right if one considers that these syllogisms do not have strict but approximate (Zadeh) validity. Typically there are only particular premises available in everyday life and one is dependent on such syllogisms. – Some rules on the usage of particular premises are given below.

Key words: approximate reasoning, fuzzy logic, possible worlds, syllogisms with particular premises, undetermined quantifiers

1. A Syllogistic Fallacy with Particular Premises

If one presents syllogisms to a lay person in the field of logic, even rather simple syllogisms to a well-educated lay person, one will often find that the test person is not able to establish the correctness of the conclusions accurately. Fritjof Haft (Haft 1978, p. 79), among others, has conducted such experiments in seminars on legal rhetoric. Here is one of his examples:

> If some public servants are co-operative and many co-operative people are efficient, then it follows that some public servants are efficient.

Haft reports that in a seminar held for judges and public prosecutors this syllogism “was almost unanimously seen as correct”.

According to the traditionally accepted rules of syllogistic reasoning the syllogism is incorrect, as it contains two particular premises (. . . some public servants . . . , . . . many co-operative people . . .). Since Aristotle it is known: ex mere particularibus nihil sequitur.

Haft offers an explanation for the result: For the participants of the seminar, the terms “public servant”, “efficient”, “co-operative” were “emotionally charged”. And (to continue his thought) a public servant being presented with a conclusion proceeding from the pleasant premise that some public servants are co-operative –

and coming to the welcome result that some of them are efficient (Who would not recognise himself?) would only be too eager to accept this conclusion as correct.

Haft's explanation seems disturbing. Judges and public prosecutors earn their living by assessing arguments with a view to the logical coherence of a law suit, the urgency of the suspicion of a criminal offence, the stringency of evidence given. If, by a little flattery, they can be lured away from established rules that have been accepted for two and a half thousand years, one can only take a dismal view of things.

2. Some Peculiar Premises

But after all the situation may not be as bad as it seems at first glance. Haft's presumption may hold true, but perhaps not to such an extent as he imagines. There is something else to be considered, namely: in his examples, Haft did not only give a bias in regard to subject and predicate terms, but also to the logical particles. Haft talks about "many (co-operative people)". But particularity has no other meaning than "at least one" in the terms of logic.

Is this really the same thing? Are "hardly anyone", "a few", "several", "many", "nearly everyone" synonymous in logic? Haft seems to accept this as fact and in this he certainly does not stand alone. The linguistic differences in signifying a particular quantity seem to be only bubbles on the hard surface of the logical structure.

However, I have the suspicion that Haft himself is not quite sure of his own view. In any case, it seems conspicuous that he did not take his examples to the extreme: "If *many* public servants (instead of only "some") are co-operative and if *many* co-operative people are efficient . . . " – then the conclusion that at least *some* public servants are efficient would certainly be even more tempting.[1]

3. Peculiarities Making the Deduction Valid in Some Worlds

But maybe also closer to the truth? I myself believe that there is some truth to such conclusions. Intentionally, I am expressing this in a "fuzzy" way; we have now reached the area that Lotfi Zadeh, the father of fuzzy logic, was the first to explore and name: the field of "approximate reasoning" (Zadeh 1975).[2] It deals with conclusions that are not strictly but approximately correct and that play an important part in everyday life.

[1] Haft gives three examples for corresponding "false reasoning". In none of them did he raise the quantification of both premises to "many" and at the same time reduce the conclusion to "some".

[2] Fuzzy logic investigates the phenomenon of concepts often being vague and their border lines blurred. Its objective is to facilitating reflective argumentation on the grounds of unspecific but real-life assumptions, as well as solving problems of technical control, when it depends on fuzzy input or strives for softened, organic transitions. The connection between non-formal language and mathematical logic, with all its technical applications, is typical of fuzzy logic. For information on fuzzy logic applied to the law cf. (Philipps 1993a–1995b).

In borderline cases such conclusions could even be completely correct, as I will show using two examples. I will intensify the premise to “most” rather than “many” and I will add a precondition, namely that the (finite) set defined by the central term M is not larger than the subject set S (Figure 1).

Most S are M.

Most M are P.

Therefore: Some S are P.

```
S  S  S  S  S  S  S  S  S  S
         M  M  M  M  M  M  M  M  M  M
                  P  P  P  P  P  P  P  P...
```

Figure 1.

```
S  S  S  S  S  S  S  S  S  S
   M  M  M  M  M  M  M  M  M  M
      P  P  P  P  P  P  P  P  P  P...
```

Figure 2.

The graph, which can easily be generalised, shows: the conclusion is necessarily true. When I carry out one further intensification towards “almost all”, then the result is a valid conclusion concerning “most” (in our example even a little toned down version on “almost all”) (Figure 2).

Within limits[3] the M-set can be larger than the S-set. If, for example, “almost all” of the elements of an S-set, consisting of 10 elements, are M, and the M-set comprises 20 elements of which “almost all” are P, then still a “more than negligible” proportion of S are P (in our example even “most” S are P) (Figure 3).

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S S S S S S S S S S
  M M M M M M M M M M M M M M M M M M M M
      P P P P P P P P P P P P P P P P P P P ...
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Figure 3.

[3] In view of fuzzyness of everyday quantifiers, it will not be possible to establish border lines cardinally. At the end of this article I shall try to draw up some ordinal rules.

If the M-set is larger than the S-set, it makes a difference which proposition on the sets is quantified more extensively. If the statement on the M-Set in the second premise (counted by its position in the relation of transitivity) is more extensive, the conclusion is more reliable than if it is the statement in the first premise. In both of the following figures, half of all S are M, whereas almost all of the M are P. In the first syllogism (Figure 4) the conclusion that some S are P is obviously valid (by an assumed set size of 10 and 20 elements). Contrarily, in the second syllogism (under otherwise unchanged premises), “almost all” of S are M and “half” of all M are P. The conclusion is invalid (Figure 5).

```
S S S S S S S S S S
          M M M M M M M M M M M M M M M M M M M M
              P P P P P P P P P P P P P P P P P P P ...
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Figure 4.

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S S S S S S S S S S
  M M M M M M M M M M M M M M M M M M M M
                  P P P P P P P P P P P P P P P P P P P
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Figure 5.

The difference is also plausible, because the first conclusion is approximate in regard to a valid classical syllogism. (Some S are M; all M are P. Therefore some S are P.) Whereas the second conclusion is approximate in regard to an invalid syllogism (with the premises: All S are M; some M are P. Therefore: ???).

Haft’s syllogism we started out with is therefore more reliable than if we had assigned the quantifiers inversely:

> If many public servants (instead of some) are co-operative and if some co-operative people (instead of many) are efficient, then it follows that some public servants are efficient.

However, as long as both sets (S and P) are of the same size, it makes no difference which quantifier is more extensive. With the help of diagrams the interested reader might like to try this out.

4. Strict Deductions for All Possible Worlds; Approximate Ones for Ours

Frequently, we will not have any precise idea as to the size of the sets involved in the situation. On the other hand, in everyday life there usually is no reason to maintain a stipulation used in conventional logic that might be called the “condition of pessimism”. This stipulation is most easily explained using Leibnitz’s concept of “possible worlds”. According to Leibnitz different laws, even different laws of nature, can be valid in different worlds. However, there are laws which are valid in all possible worlds - and those are the laws of logic. Logical laws would even be valid in a world that is the worst possible for argumentation. But do we have reason to assume that we ourselves live in the worst of all argumentative worlds? E.g. in a world unfavourable to arguments, there would be no place for jurisprudence as we know it, because its conclusions are certainly not strictly logical.

In the graph I introduced above, I used the precondition of logical pessimism. I spaced the strings of symbols as far apart as possible, making them as little congruent as possible, so that as few elements of S are allocated to elements of P as possible – in this way only unavoidable conclusions can be drawn. If I had even chosen "half" instead of "most" for quantifier a necessarily true conclusion would not have been possible. "Half" constitutes the border line case (Figure 6).

```
S S S S S S S S S S
          M M M M M M M M M M
                    P P P P P P P P P P...
```

Figure 6.

```
S S S S S S S S S S
 M  M  M  M  M  M  M  M  M  M
    P     P     P     P     P     P     P ...
```

Figure 7.

Not a necessarily true conclusion - but a sensible one, perhaps? Actually we have to ask ourselves: why should the M-elements with regard to the S-elements as well as the P-elements with regard to the M-elements be distributed in such an incongruent way? In the laboratory of logic we have to proceed on this prerequisite, but in everyday life the assumption is rarely realistic. We do not need to restrict ourselves to forms of conclusions that are valid in all possible worlds to come to sensible conclusions in our actual world of here and now.

If half of all S are M and half of all M are P, then the distribution could look something like this. One could conclude that a quarter of all S are P: not a negligible proportion (Figure 7).

In order to replace pessimism with realism I now replaced the extreme distribution with an even distribution. Admittedly the simple and strictly determined sequence in the graph is a didactic stylisation. What we can expect is merely this: If – statistically – every other S is a M and every other M is a P, then – from a statistical point of view – every fourth S is a P. The larger the sets in the premises, the more reliable the result.

On principle one would get the same result if I had chosen the undetermined quantifier "many" instead of the definite quantifier "half". If many S are M and many M are P, it usually follows that “more than an negligible proportion” of S are P.

Haft’s judges and public prosecutors might have been guided by such reasoning. They thought they should judge the conclusions in view of their common sense, presumably not realising that they were exposed to a test of Aristotelian logic. In their way they were right.

5. Approximate Deductions and the Question of the World’s Uniformity

Now the question arises: What justification is there to assume an even distribution? Does uniformity exist throughout the world? This is a long-standing question which has been hotly debated again and again in view of the permissibility of a conclusion by induction. Since all these attempts have failed, it can be concluded that there is no justification in an assumption of uniformity from the point of view of ontology. Depending on one’s perspective, the world is both uniform and non-uniform. If we can assume uniformity in everyday life, this is only because in the part of the word in which we feel at home, we are familiar with the regularities. The exceptions we noticed are often reinterpreted by us as new rules. The assumed uniformity is no more than a presumption and a shifting of the burden of proof: I am permitted to assume uniformity – but only as long as I have no grounds to doubt that in the case in question things could differ from the usual. For this another of Haft’s examples is suitable:

> Many jurists are outstanding writers.
>
> Many outstanding writers are admired.
>
> Therefore: Many jurists are admired.

We are left to wonder what the participants of Haft’s seminar thought about this syllogism.[4] Probably they did not find it very convincing. For the premises may

[4] It is certainly not conclusive. Concerning the quantifier “most”, Drösser (1994, p. 74) gives this pretty example for the opposite:

> Most inhabitants of Berlin are Germans.
>
> Most Germans live to the west of the river Elbe.
>
> Therefore: Most inhabitants of Berlin live to the west of the river Elbe.

But Drösser also has “the feeling that somehow the conclusion is after all ‘mostly’ a correct one”. The fallacy contained in this argument would be apparent even to somebody who knows no more about Germany than that Berlin is a City – someone in short who has only a general structural idea that the S-elements are clustered and not more or less evenly distributed.

sound flattering (incidentally may even also be true) and the form of reasoning may be approximately accurate, however the conclusion appears to be implausible.

Most of the judges and public prosecutors will be realistic enough to know that jurists are too unpopular to be found admirable by many people. As for being admired, the jurists-set is an exception to the excellent-writers-set. Exceptionally, the reasoning is incorrect in this case.

Conclusions resulting from particular premises are of a presumptive kind only (setting aside the special instances mentioned above). They might have to be revised. This is done as follows:

In case the result of a syllogism with particular premises appears to be questionable, one has to examine the content of the intersection of S and P. Are there really "many" or "most" etc. elements of the Subject-set (depending on the requirements of the quantifier) contained in the Predicate-set? The approximate conclusion has to be discarded if this is not the case.

If the combination of the S-sets and P-sets appears suspicious for reasons of content (like the combination of "being a jurist" and "being admired" in the example), the sets have to be treated as being incongruent in the same way that they would be treated according to the general principle of pessimism. Then approximate reasoning reverts to the border line case of strict reasoning.

6. The Necessity of Relying on Particular Premises in Everyday Life

I have attempted to show that in the world we inhabit it is possible and sensible to draw conclusion ex mere particularibus. Actually, we depend on such conclusions. In everyday life a sentence containing the quantifier "all" is hardly ever correct; the phrase "There is no rule without exception" holds true. In saying so, we are referring to common sense rules, not mathematical theorems or statements as evident as "All men at this table are wearing neckties".[5]

On the other hand sentences about the quantifier "some" are practically always correct in everyday life, if one accepts the assumption of traditional logic that the quantifier "some" embraces everything from "one case" to "all cases", i.e., all but "no case".[6] In everyday life, one will almost always find some pertinent instance. That is why beside the saying "There is no rule without exception" it is said: "Nothing exists that doesn't exist". (In German: "Es gibt nichts, das es nicht gibt".)[7]

If a sentence containing one of the two quantifiers is virtually never correct whereas a sentence with the other one is practically always correct, then in every day life the traditional "crisp" syllogism based on a well balanced interplay of

5 Therefore by realistic interpretation the sentence is not the paradox discussed in some text books on logic.

6 In accordance with everyday usage and older logic I read the term "all" as presupposing the existence of at least one case; the set is not empty.

7 Here too, the paradox is only a rhetorical one.

the quantifiers "all" and "some" is of little use. Nonetheless, we have to draw conclusions constantly in everyday life in order to act. Such conclusions are only possible from particular premises, but then have to be weaker, approximate.[8]

I have tried to compile some rules of such an everyday logic. The rules do not contradict traditional logic but are an approximation to it. They approach logical correctness or, looking at it differently, cautiously stray from it.

These rules are realistic and for example should be practised at seminars of rhetoric by judges, lawyers, and public prosecutors.

7. Some Rules for Approximate Syllogisms with Particular Premises

1. The stronger the terms of quantification in the premises are – approaching "all" ("many", . . . "nearly all") – and the weaker the term of quantification in the conclusion is ("some", . . . "a few"), the more reliable the conclusion will be.

2. The smaller the amount by which the size of the set defined by the intermediate term (M-set) exceeds the size of the subject term (S-set), the more reliable the conclusion will be.

3. There is a difference as to which of the statements on the sets is quantified more extensively: If it is the statement about the M-set (in the second premise, counted by position in the relation of transitivity) that is more extensive, then the conclusion will be more reliable than when the statement about the S-set is more extensive.

4. If the elements of sets named in the premises are clustered and not evenly distributed, the conclusion will be less reliable.

References

Drösser, Ch. 1994. *Fuzzy Logik: Methodische Einführung in krauses Denken.* Hamburg: Rowohlt.

Haft, Fr. 1978, *Juristische Rhetorik*. Freiburg/München: Karl Alber.

Murphree, W.E. 1993. Expanding the traditional syllogism. *Logique & Analyse* 141–142, 105–120.

Philipps, L. 1993a. Vague legal concepts and fuzzy logic: An attempt to determine the required period of waiting after traffic accidents. *Informatica e Diritto* 37–51.

Philipps, L. 1993b. Kompensatorische Verknüpfungen in der Rechtsanwendung – ein Fall für fuzzy logic. In M. Martinek (ed.), *Vestigia Iuris: Festschrift für Günther Jahr*. Tübingen: J.C.B. Mohr.

Philipps, L. 1994. Ein bißchen fuzzy logic für Juristen. In M-T. Tinnefeld et al. (ed.), *Institutionen und Einzelne im Zeitalter der Informationstechnik*. München: Oldenbourg.

Philipps, L. 1995a. Just decisions using multiple criteria or: Who gets the Porsche? An application of Ronald R. Yager's fuzzy logic method, *Proceedings of the Fifth International Conference on Artificial Intelligence and Law*. College Park, MD: University of Maryland, pp. 195-200.

Philipps, L. 1995b. Eine Theorie der unscharfen Subsumtion – Die Subsumtionsschwelle im Lichte der fuzzy logic, *Archiv für Rechts- und Sozialphilosophie* **81**, 405–417.

Zadeh, L. A. 1975. Fuzzy Logic and Approximate Reasoning, *Synthese* **30**, 407–428..

[8] The above mentioned explanations touch in some points on the extremely interesting paper by (Wallace E. Murphree 1993). In fact Murphree changes the meaning of particular premises to sentences containing "all" with exceptions as follows: "At least all S are M with the exception of 10". The "numeric" syllogism reached in this way is crisp. But the formulation of its premises requires assumptions which will only be rarely met in everyday life.