



Ownership: A case study in the representation of legal concepts

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Abstract. This article is an exercise in computational jurisprudence. It seems clear that the field of AI and Law should draw upon the insights of legal philosophers, whenever possible. But can the computational perspective offer anything in return? I will explore this question by focusing on the concept of OWNERSHIP, which has been debated in the jurisprudential literature for centuries. Although the intellectual currents here flow mostly in one direction – from legal philosophy to AI – I will show that there are also some insights to be gained from a computational analysis of the OWNERSHIP relation. In particular, the article suggests a computational explanation for the emergence of abstract property rights, divorced from concrete material objects.

1. Introduction

I have struggled with the concept of OWNERSHIP for a number of years. In my earliest work on TAXMAN (McCarty 1977), “own” was treated as a primitive predicate and the representation of corporate transactions was constructed almost entirely out of elementary changes in the ownership relation. In TAXMAN II (McCarty 1980; McCarty and Sridharan 1981), it became necessary to decompose the ownership relation further, at least for the ownership of corporate securities, and a more primitive representation in terms of the rights of stockholders and bondholders was proposed. This led to the development of a logic for reasoning about “permissions and obligations” (McCarty 1983, 1986), which then turned out to be of independent interest. More recently, the lessons learned in this earlier work have been applied in my *Language for Legal Discourse* (McCarty 1989). Although LLD is intended to be used in a number of applications, e.g., Schlobohm and McCarty (1989), it still bears the imprint of the initial problem that led to its development: the representation of the legal concept of ownership (McCarty 1995).

In all of this work, I have tried to make use of the insights of legal philosophers. The concept of ownership has been analyzed by every classical philosopher who wrote on the subject of law, from Locke and Bentham to Kant and Hegel. Likewise, in the twentieth century, interpretations of the concept of ownership have run the gamut, from Wesley Hohfeld (1913, 1917) to Richard Posner (1973). Overall, the

jurisprudential literature is rich and diverse. Can the computational perspective offer anything in return? I will explore this question in the present article.

Section 2 summarizes the history of OWNERSHIP in Anglo-American law, and raises questions about the extent to which this history has been influenced by linguistic and cognitive factors. (I suspect that a similar story could be told about continental European law, but I will leave that to someone more familiar with the civil law tradition.) Section 3 then reformulates these questions in computational terms, using the representational devices of *LLD*.

The main puzzle is this: The traditional and commonsense view is that ownership is a relation between a *person* and a *thing*, whereas the modern and sophisticated view is that ownership is a *bundle of rights*. These views coexist without difficulty when the thing owned is a concrete material object. But why do we talk about the *ownership of abstract objects*, such as “debts” and “stocks” and “bonds”? Here, it seems, there are rights and duties – and nothing more. Why do we pretend that there is also an abstract “thing” in these situations that is part of an ownership relation?

The article suggests a computational answer to this question, in Section 3, using the example of a simple “claim”, and then extends the computational analysis, in Section 4, to the more complex example of stocks and bonds. Section 4 also raises some broader questions by developing the computational thesis within the framework of a famous early lecture by H. L. A. Hart, *Definition and Theory in Jurisprudence* (1984). The hope is that this juxtaposition of a computational and a jurisprudential perspective will stimulate further thought along these lines.

2. A conceptual history

There are roughly three stages in the evolution of the concept of OWNERSHIP in Anglo-American law:¹

At an early stage, the concept as we know it today did not exist. J. C. Smith points out that the word “owner” first appears in the English language in 1340, and the word “ownership” first appears in 1583 (Smith 1976, p. 214). The term describing the legal relationship between a person and a thing in early English law was “seisin”, and it corresponded more closely to the concept of physical possession than to the concept of ownership. If a person was “disseised” of his lands, he could maintain an action to recover possession, but in the meantime the “disseisor” could convey the lands to someone else or pass them on to his heirs as part of his estate. If a person was “disseised” of his goods, his rights were even more limited, since he had no action to recover possession of the goods but only an action for damages (Ames 1913). Such was the state of English law in the middle of the 13th century. At a later stage, the remedies against wrongful dispossession were substantially expanded, and the concept of seisin began to approach the modern concept of ownership. Honoré describes this development as follows:

To have worked out the notion of *having a right* to as distinct from merely *having*, or, if that is too subjective a way of putting it, of rules allocating things to people as opposed to rules merely forbidding forcible taking, was a major intellectual achievement. Without it society would have been impossible. (Honoré 1961)

This evolutionary process reached its peak by the middle of the 18th century, when Blackstone could describe property as “that sole and despotic dominion which one man claims and exercises over the external things of the world, in total exclusion of the right of any other individual in the universe” (Blackstone 1791, vol. II, p. 2).

A third stage, quite familiar today, involves the extension of the concept of ownership to abstract objects. Thus a “debt” can be viewed as a form of property, which can be bought and sold in the same way as an automobile or a house. Sophisticated extensions of this basic idea have produced the wide range of securities issued by modern corporations: “stocks” and “bonds” and “convertible subordinated debentures”, to name just a few. Patent law and copyright law have extended the concept of ownership in another direction, treating various intellectual creations (“inventions” and “expressions”, respectively) as if they, too, were abstract objects. A parallel development in the modern era can be seen in a split between lawyers and nonlawyers about the nature of the concept of ownership itself. The commonsense concept of ownership today is still reminiscent of Blackstone, i.e., it is a relationship between a *person* and a *thing*. But to lawyers and other technical specialists, ownership is generally understood as a “bundle-of-rights”. Each “stick” in the “bundle” is a distinct “right” in the ownership relation, which can be split off and transferred separately. Using the bundle-of-rights metaphor, of course, it does not make much difference whether the thing owned is a concrete physical object or an intangible abstraction, since the focus of attention is now on the “sticks”.

To understand these modern developments, it is helpful to analyze the concept of ownership using Hohfeldian terminology (Hohfeld 1913, 1917; Lindahl 1977). Wesley Hohfeld’s “fundamental legal conceptions” are shown in Figure 1, with minor modifications.² The concepts in the first column are all “rights” in a general sense, i.e., they are legal relations between two persons viewed from the perspective of the person who is, in general, advantaged by the relationship. Thus we say that *X* has a *claim* that *Y* either do or refrain from doing a particular action, or we say that *Y* is at *liberty* either to do or refrain from doing a particular action insofar as *X* is concerned. The column marked “Correlatives” expresses the same relationship from the perspective of the other person. For example, if *X* has a *claim* against *Y*, then *Y* has a *duty* to *X*, and vice versa. Roughly speaking, the column marked “Opposites” expresses the absence of the relationship in the column marked “Rights”. This is obvious in the case of a *no-claim* between *X* and *Y*, which holds just in case there is no claim in existence between *X* and *Y* with respect to a particular action α . The opposite of a liberty must be stated more carefully, however. For example, the opposite of *Y*’s liberty to do a particular action α , insofar as *X* is concerned, is the existence of a duty on *Y* *not* to do the action α .

Rights	Correlatives	Opposites
claim	duty	no-claim
liberty	no-claim	duty
power	liability	no-power
immunity	no-power	liability

Figure 1. Hohfeld's fundamental legal conceptions.

The last two rows of the table in Figure 1 represent a different kind of legal relation, in which the actions themselves involve changes in other relationships. Thus, we say that *X* has a *power* over *Y* if *X* is able to bring about a change in some legal relationship affecting *Y*. (Note that *Y* could be the same person as *X*, so that *X* could alter his or her own legal relationships.) The correlative of a power is a *liability*, i.e., a situation in which *Y*'s legal relationships are subject to change by *X*, and the opposite of a liability is an *immunity*. Thus we say that *X* has a power over *Y* with respect to a particular change in the relationship *R* if and only if *Y* has a liability to *X* with respect to this particular change in the relationship *R*, and if this is not the case then we say that *Y* has an immunity from such a change in *R* being brought about by *X*. Note that powers and immunities are generally advantageous, and are therefore loosely referred to as "rights", but liabilities are not always disadvantageous. For example, the beneficiary of a trust is subject to the *liability* of receiving money from the trustee.

Using this analytical vocabulary, we can now be more precise about the concept of ownership. Let us, for the moment, consider only the ownership of concrete physical objects. Then the *owner* of an *object* has the following Hohfeldian rights:

- A *claim* against other persons to exclusive physical control of the object, i.e., other persons would have a *duty* not to use the object in any way, or take any actions that would harm or destroy it.
- A *liberty* to use (or consume, or destroy) the object.
- A *power* to transfer all (or some) of these rights to another person.
- An immunity from the involuntary expropriation of these rights by other persons.

This is roughly the analysis of ownership proposed by Hohfeld in his original articles. Subsequent authors, such as Honoré (1961), have suggested a much longer list of the "incidents" of ownership, including such negative incidents as the *duty* that the object not be used to cause harm and the *liability* that the object might be seized to satisfy a judgment. But the preceding list is sufficient for our present purposes.

It is a matter of debate just how important Hohfeld's analysis was for the actual development of the law. Arthur Corbin, writing a foreword to the republication of Hohfeld's articles in 1964, admits that Hohfeld's terminology did not catch on, but suggests that his analytical method had a subtle influence on legal thought

(Hohfeld 1964). He tells the story of Williston, who drafted the American Law Institute's (First) *Restatement of the Law of Contracts*, checking to make sure that his analysis was consistent with Hohfeld's, and he points out that the first chapter of the American Law Institute's *Restatement of the Law of Property* was explicitly drafted in a Hohfeldian style. "That chapter must have had some influence on judicial and professorial thought", Corbin writes.

Corbin thus raises a question, in a narrow context, that actually has much broader scope: Does the way we think about legal concepts make a difference? Does it matter whether ownership is understood as a relation between a "person" and a "thing" or as an abstract "bundle-of-rights"?³ Do such conceptualizations have an impact on the development of the law? Or does the causal explanation go the other way?

These questions are also raised by two contemporary legal scholars, Thomas Grey and Charles Donahue, Jr., in their contributions to a *Nomos* symposium on *Property* in 1977. Grey writes about the disintegration of property in the modern world, arguing that the term "property" no longer denotes a coherent concept and suggesting that this fact is politically significant. Part of Grey's analysis is simply the observation, noted above, that the concept of ownership had evolved from the time of Blackstone to the time of Hohfeld. This evolution was internal to the development of capitalism, Grey suggests. As the industrial economy matured, it became necessary to divide and rearrange the simple ownership of objects into more and more complex forms. But these new forms were primarily the constructions of lawyers, and their work was greatly facilitated by the analysis of property as a "bundle-of-rights".

The "bundle-of-rights" conception of property appears in well-articulated form for the first time (insofar as I have discovered) in Wesley Hohfeld, "Some Fundamental Legal Conceptions as Applied in Judicial Reasoning", 23 *Yale Law Journal* 16 (1913). Thereafter, it became part of the conceptual stock-in-trade of the legal realist movement, often with the strong implication that "private" and "public" property were not as different as traditional property theory would suggest. (Grey 1980, p. 85, n. 40)

Ultimately, Grey argues, the Hohfeldian analysis subverted the concept of property itself, and weakened the moral foundations of capitalism.⁴

Donahue's thesis is quite different, but it raises some of the same questions. Although Donahue acknowledges that "the concept of property itself is in trouble" (Donahue 1980, p. 28), he also notes an "agglomerative tendency" throughout history that cuts against the pressures for fragmentation. Thus, in both classical Roman law and medieval English law, Donahue observes a tendency to "agglomerate" the Hohfeldian rights in a single legal person, preferably the one currently in possession, and to treat the rights of other individuals as limited exceptions. This was true, Donahue claims, even before the rise of liberal political theory in the 18th century. Moreover, there is no obvious social explanation for the tendency. Instead,

Donahue proposes a conceptual explanation. The tendency began as an allocation of the burden of proof, he suggests. Then:

As the need for a category arose to describe the sum of the rights, powers, and privileges that an individual could have with respect to a thing, we chose the noun derived from the adjective that means “own”. The category at once described the concept and also the tendency. As time went on, the tendency took on an independent life. (Donahue 1980, p. 45)

Is this agglomerative tendency strong enough today to overcome the tendency towards disintegration noted by Grey? Donahue tries to answer this question by analyzing trends in the use of the word “property” by state appellate courts over a 20-year period, but his results are inconclusive.

From our perspective, the interesting point here is that Grey and Donahue (and also Corbin, to a lesser extent) are engaged in a debate about *cognitive phenomena*. They are seeking explanatory power from a study of the way lawyers use legal concepts in different historical periods. But explanations of this sort have always been greeted with skepticism. Would they be more persuasive if they were reformulated in computational terms? We will consider this question in the following section.

3. A computational reconstruction

I will assume that the reader is generally familiar with my *Language for Legal Discourse (LLD)*, as described in McCarty (1989) and applied in Schlobohm and McCarty (1989) and McCarty (1995). However, several features of the language are needed for an understanding of the material in this section, and I will review these briefly.

At its most basic level, *LLD* provides a mechanism for the representation of *objects* and *relationships*, each of which can be arranged in a sort hierarchy. For example, we could have a sort Actor with subsorts Person and Corporation. One important feature is the distinction between *count terms* and *mass terms*. Mass terms, e.g., Gasoline and Cash, can have quantitative measures, e.g., Volume and Value, attached to them. A quantitative measure expresses a relationship between (i) an individual mass; (ii) a reference mass; and (iii) a number, as in the following example:

```
(Value -
  {object (Cash 'C-1)}
  {unit (Dollar 'D-1)}
  {quantity (Number 1000)}}),
```

This syntax allows the arguments of a relation to be inverted, so that we can talk about “the Cash 'C-1 which is the *object of* a measure of Value with a reference mass of a Dollar and a quantity of 1000”. In other words: one thousand dollars.

It is useful to distinguish two types of definitions in *LLD*, even though both are written in the same way. One type is the traditional definition *per genus et differentiam*. For example, assume that we have the sort *Corporation* and we want to define the subsort *SmallBusinessCorporation*. We might encode this as follows:

```
(SmallBusinessCorporation C)
← (Corporation C)
AND
"C satisfies IRC Section 1371",
```

where we have simply abbreviated the conditions listed in §1371 of the Internal Revenue Code, assuming that they could, in fact, be written out in full. Notice that the variable *C* appears here in both the *definiens* and the *definiendum*, and thus no new objects are created by this definition. The second type of definition does create a new object, or a new relationship. An example is the concept of “control” in §368(c) of the Internal Revenue Code. Assuming that the relations *Own* and *Issued* have already been defined, we could define a simplified version of the concept of *Control* as follows:

```
(Control C1 {subject (Actor A)} {object (Corporation C)})
← (Own O1 {subject (Actor A)} {object (Stock S)})

AND

(Issued I1 {subject (Corporation C)} {object (Stock S)})
```

In this example, the variable *C1* represents an instance of the relation *Control*, i.e., it is a new *relationship*.⁵

The representation of objects and relationships only goes so far, however, and *LLD* also provides mechanisms for the representation of *events* and *actions*. Elementary events are represented by *statechanges*, as in Figure 2. Complex events are defined by Horn clauses similar to those defining complex relations, except that they presuppose a *linear order* on the underlying time points and allow order relations to appear as part of the definitions.⁶ Finally, an *action* is a relationship between an actor and an event, which may be either elementary or complex.

In all of these examples, of course, we have been assuming that *Own* (and *Corporation* and *Stock*) are either primitive concepts, as in McCarty (1977), or concepts that have previously been defined. But for the present article, this begs the question. The objective here is to define the concept of *OWNERSHIP* itself. The two types of definitions discussed above correspond roughly to the *terminological* component and the *assertional* component, respectively, in various other knowledge representation systems (Brachman et al. 1985). Are these two

```

(StateChange -
  {relation1
    (Own -
      {subject (Actor A1)}
      {object (Property P)}})
  {relation2
    (Own -
      {subject (Actor A2)}
      {object (Property P)}})
  {time1 (Time T1)}
  {time2 (Time T2)})

```

Figure 2. An example of a *statechange*.

components sufficient for the definition of OWNERSHIP? I will argue in Section 4 that additional modes of definition are required, but first we need to develop some additional machinery.

The basic strategy for a computational analysis of the ownership relation is: (1) to formalize Hohfeld’s system in *LLD*; and (2) to write down the incidents of ownership in the formalized Hohfeldian language. To do this, we need to examine the modalities over actions that are available in *LLD*. The most prominent modalities are deontic: *permitted*, *forbidden*, *obligatory*. For expository purposes, we will write these in two ways. When a short form is needed, we will use (roughly) the notation in McCarty (1983): $\mathbf{P}\langle\phi \mid \alpha\rangle$, $\mathbf{F}\langle\phi \mid \alpha\rangle$, $\mathbf{O}\langle\phi \mid \alpha\rangle$. For example, $\mathbf{O}\langle\phi \mid \alpha\rangle$ means: “Under the condition ϕ , the action α is obligatory”. When we want to write out a complete representation of the conditions and actions, however, we will use *LLD* syntax:

```

(Permit 'PE-1
  {condition . . . }
  {action . . . })

(Forbid 'FO-1
  {condition . . . }
  {action . . . })

(Oblige 'OB-1
  {condition . . . }
  {action . . . })

```

Recall that $\mathbf{P}\langle\phi \mid \alpha\rangle$ is a “free-choice” permission, meaning “under the condition ϕ , all ways of doing the action α are permitted”. If we needed a weaker form of permission – “some way of doing the action α is permitted” – we would use the modality $\neg\mathbf{F}\langle\phi \mid \alpha\rangle$. However, it turns out that we also need a modality over actions that is structurally similar to “not forbidden”, but with an interpretation in terms of

the agent's *ability* to perform the action rather than the *permissibility* of the action.⁷ We call this the *enabled* modality, and write it as follows:

```
(Enable 'EA-1
  {condition . . . }
  {action . . . })
```

When a short form is needed, we will write it as $\mathbf{E}\langle\phi | \alpha\rangle$.⁸

This is all we need to represent Hohfeld's fundamental legal conceptions. Suppose $\alpha(Y, X)$ represents an action performable by Y that happens to benefit X . Then $\mathbf{O}\langle\phi | \alpha(Y, X)\rangle$ represents the fact that Y has a duty to do $\alpha(Y, X)$, assuming ϕ is true, and this means that X has a (conditional) claim against Y for the performance of α . Suppose $\beta(Y, X)$ represents an action performable by Y that happens to be detrimental to X . Then $\mathbf{F}\langle\phi | \beta(Y, X)\rangle$ represents the fact that Y has a duty *not* to do $\beta(Y, X)$, assuming ϕ is true, and this means that X has a (conditional) claim against Y that β *not* be performed.⁹ On the other hand, $\neg\mathbf{F}\langle\phi | \beta(Y, X)\rangle$ tells us that Y is at liberty to perform β , and $\neg\mathbf{O}\langle\phi | \alpha(Y, X)\rangle$ tells us that Y is at liberty not to perform α . For the remaining Hohfeldian categories, assume that $\Delta R(X, Y)$ represents an action in which X brings about a change in the legal relation R affecting Y . For example, such an action could be written using the *statechange* formalism illustrated above. Then $\mathbf{E}\langle\phi | \Delta R(X, Y)\rangle$ tells us that X has a power over Y with respect to ΔR , and $\neg\mathbf{E}\langle\phi | \Delta R(X, Y)\rangle$ tells us that Y has an immunity from X with respect to ΔR .¹⁰

Let us now return to the concept of OWNERSHIP. We have just done the easy part: the representation of a "bundle of rights" in Hohfeldian terminology. The hard part is to represent particular objects, and the particular actions that can occur. Some of the distinctions here are legally significant – e.g., the distinction between reality and personalty, or between movables and immovables – and some of the distinctions are just plain matters of common sense. For example, consider the distinction between count terms and mass terms, mentioned earlier. For count terms, it is usually sufficient to consider actions that apply only to the object as a whole. But for mass terms, the most natural way of using the object might be to split off various parts of the mass, and to apply different actions to different parts. Our action language must be capable of representing this fact, and the distinction between count actions and mass actions will then percolate upwards into our representation of the Hohfeldian bundle of rights.

In general, the bundle of rights associated with *tangible* objects is at least as complex as the tangible world itself. On the other hand, the bundle of rights associated with *intangible* objects is often much simpler. Let us see how the ownership of intangible objects would be represented.

First, consider a specific obligation to deliver a specific quantity of goods on a specific date. For example, Armstrong might have a contractual obligation to deliver 5,000 bushels of wheat to Brubaker in Chicago on September 1, 2000. (For

```

(Oblige 'OB-1
  {condition TRUE}
  {action
    (Deliver -
      {agent (Person 'Armstrong)}
      {object (Wheat W1
        {objectof
          (Volume -
            {unit (Bushel 'B-1)}
            {quantity (Number 5000)}}))}
      {recipient (Person 'Brubaker)}
      {location (City 'Chicago)}
      {time1 (Time '1-September-2000)}
      {time2 (Time '2-September-2000)}}))}

```

Figure 3. Armstrong's obligation to deliver wheat to Brubaker.

simplicity, we will assume that the purchase price for the wheat has already been paid in full.) This obligation might be represented as shown in Figure 3. The action *Deliver* is tangible and complex, of course, and we will simply assume that it has been properly represented in *LLD*. Thus, Brubaker has a *claim* for the delivery of the wheat. In early English law, contract claims could not be assigned to third parties, but by the end of the 18th century these rules had changed. Today, for example, §2-210 of the Uniform Commercial Code explicitly allows the assignment of rights and the delegation of duties, with certain exceptions, none of which apply to Armstrong and Brubaker. Thus, Brubaker's claim is *assignable*. The right to assign a claim arising out of contract is a Hohfeldian *power*. The question is: How should such a power be represented?

The basic mechanisms are already available in *LLD*. We first create a *StateChange* in which *relation1* is the obligation OB-1 and *relation2* is the same as OB-1 but with the recipient, Brubaker, replaced by an anonymous actor, A. We then stipulate that this is an action that Brubaker is able to perform, using the *Enable* modality. Now, suppose Brubaker actually carries out this action, with A = Cadbury. The result will be a claim by Cadbury for the delivery of the wheat. However, we might now want to say that Cadbury has the power to assign the claim, to Dawkins, say, who would then have the power to assign the claim to Edelman, and so on. We cannot write this out, sequentially, for all finite sequences of assignments. Instead, we would like to find a representation in which the power to transfer is itself one of the rights transferred.

Since the solution to this problem is familiar, I will present it first and ask questions about it later. Suppose we define a new kind of property – which might as well be called a *Claim* – and then write an initial assertion that Brubaker owns a particular instance of this new kind of property. For example, we could assert that Brubaker owns *Claim 'C-34* in *State 'S-2*, as shown in Figure 4. We then replace the unconditional obligation OB-1 with a conditional obligation OB-2,

```

(State 'S-2
  {relation
    (Own -
      {subject (Person 'Brubaker)}
      {object (Claim 'C-34)}})
  {time (Time '15-June-2000)})

(Oblige 'OB-2
  {condition
    (Own -
      {subject (Actor A1)}
      {object (Claim 'C-34)}})
  {action
    (Deliver -
      {agent (Person 'Armstrong)}
      {object (Wheat W1
        {objectof
          (Volume -
            {unit (Bushel 'B-1)}
            {quantity (Number 5000)}}))}})
    {recipient (Actor A1)}
    {location (City 'Chicago)}
    {time1 (Time '1-September-2000)}
    {time2 (Time '2-September-2000)}})})

```

Figure 4. Defining a Claim and a conditional obligation to deliver the wheat.

which is also shown in Figure 4, and which states that Armstrong is obligated to deliver the wheat to any Actor A1 who owns Claim 'C-34. Clearly, if Brubaker's ownership of claim C-34 persists from June 15 to September 1, then OB-2 would have the same effect as OB-1. However, there is also a general rule governing the ownership and transfer of property, as shown in Figure 5. This rule states that any Actor A1 who owns Property P1 has the *power* to transfer the ownership of P1 to an Actor A2, and since the Property P1 can always be instantiated to the Claim 'C-34, Brubaker would be able to use this rule to transfer his claim to Cadbury, who could transfer it to Dawkins, who could transfer it to Edelman, etc. Whoever ends up as the "owner" of C-34 on September 1, 2000, is the person to whom Armstrong is obligated to deliver the wheat.

This solution works because of the persistence of the object C-34 through time while the ownership relation for C-34 is allowed to change. Are there solutions that do not involve the creation of new objects? One possibility is to use the existing objects in the original obligation OB-1 to define a new relationship corresponding to the claim. For example, the English word "owe" could be used to describe this new relationship. (It is interesting to observe that "owe" and "own" have a similar etymology.) We could thus say that "Armstrong owes 5,000 bushels of wheat to

```

(Enable 'EA-2
  {condition
    (Own -
      {subject (Actor A1)}
      {object (Property P1)}})
  {action
    (Action -
      {agent (Actor A1)}
      {event (StateChange -
        {relation1
          (Own -
            {subject (Actor A1)}
            {object (Property P1)}})
          {relation2
            (Own -
              {subject (Actor A2)}
              {object (Property P1)}})}})
      {time1 (Time T1)}
      {time2 (Time T2)}})

```

Figure 5. A general rule governing the ownership of property.

Brubaker deliverable in Chicago on September 1, 2000” just in case the obligation OB-1 holds. Now replace Brubaker with a variable, and rewrite the enabled action as shown in Figure 6. Finally, in the state corresponding to 15-June-2000, assert the proposition “Armstrong owes 5,000 bushels of wheat to Brubaker deliverable in Chicago on September 1, 2000”. The net result is the same as before: Brubaker would be able to transfer his claim to Cadbury, who could transfer it to Dawkins, etc.

However, there are two problems with this representation:

1. We would have to write a separate *Enable* statement for each particular instance of “owe”. This is bad enough when each instance of “owe” has the same set of arguments: *agent*, *object*, *recipient*, *location*, *time*. But a different action inside OB-1 could lead to a completely different version of the “owe” relation, with a different set of arguments.
2. The instantiation of “owe” is individuated only by the objects it refers to: Armstrong, 5000-bushels-of-wheat, Chicago and 1-September-2000. If Armstrong happened to enter into a contract with someone else for the delivery of the same quantity of wheat in the same city on the same date, these obligations could easily become confused with one another. The *Enable* statement would imply that both obligations were assignable, for example, but this might not have been the intention of the parties.

We can solve the second of these problems by using OB-1 itself as the persistent object. The basic idea is to *reify* the contractual obligation to deliver the wheat

```

(Enable 'EA-3
  {condition
    ''Armstrong owes 5,000 bushels of wheat to''
    A1
    ''deliverable in Chicago on September 1, 2000''}
  {action
    (Action -
      {agent (Actor A1)}
      {event (StateChange -
        {relation1
          ''Armstrong owes 5,000 bushels of wheat to''
          A1
          ''deliverable in Chicago on September 1, 2000''}
        {relation2
          ''Armstrong owes 5,000 bushels of wheat to''
          A2
          ''deliverable in Chicago on September 1, 2000''}
        {time1 (Time T1)}
        {time2 (Time T2)}}))

```

Figure 6. A solution using a defined relationship.

in Chicago on September 1, 2000, while treating the recipient as a free variable. We could then write the 'Enable' statement with the obligation OB-1 lexically embedded inside it, as shown in Figure 7. With this representation, there could be no confusion in the analysis of the enabled action, since OB-1 would be distinct from every other obligation that happened to refer to the same objects. Essentially, OB-1 plays the same functional role here as C-34. The only difference lies in the response to the first problem listed above. If, in fact, we needed to write special rules for the assignment of special obligations, then the use of a reified obligation OB-1 would be a good solution. However, for most purposes, the construction of a new object, C-34, provides a better level of abstraction. The persistent structure of the claim can be encoded in a conditional obligation such as OB-2, while the more transient identity of the claimant can be encoded in the ownership relation.

Furthermore, once we have accepted the idea of treating an abstract claim like a concrete object, additional possibilities emerge. For example, since wheat is a mass term, we can "split" Armstrong's obligation to deliver 5,000 bushels of wheat into an obligation to deliver 2,000 bushels to Brubaker and 3,000 bushels to Cadbury. It would be cumbersome to write out all such possibilities, of course, for every obligation and for every conceivable action involving mass terms. Instead, we could treat claim C-34 as a mass term itself, and write a small number of general rules to enable a "split" in the ownership of a claim. The obligation OB-2 would be virtually unchanged, except for the fact that it would now include a measure of the mass of C-34 that would be correlated with the volume of wheat delivered. Once again, by factoring out the persistent structure of the claim from the transient (and "split-table") structure of the ownership relation, we have simplified our representation substantially.

```

(Enable 'EA-4
  {condition
    (Oblige 'OB-1
      {condition TRUE}
      {action
        'Armstrong delivers 5,000 bushels of wheat to'
        A1
        'in Chicago on September 1, 2000'}})}
{action
  (Action -
    {agent (Actor A1)}
    {event (StateChange -
      {relation1
        (Oblige 'OB-1
          {condition TRUE}
          {action
            'Armstrong delivers 5,000 bushels of wheat to'
            A1
            'in Chicago on September 1, 2000'}})}
      {relation2
        (Oblige 'OB-1
          {condition TRUE}
          {action
            'Armstrong delivers 5,000 bushels of wheat to'
            A2
            'in Chicago on September 1, 2000'}})}
      {time1 (Time T1)}
      {time2 (Time T2)}})}

```

Figure 7. A solution using a reified obligation.

The reader should recognize this strategy as the same strategy that led to the invention of corporate securities. Stocks and bonds are intangible mass terms, with relatively simple rules governing the split and the transfer of ownership. But the persistent structure of the Hohfeldian rights attributed to stockholders and bondholders is very complex. The alternative solutions that we examined in the case of an obligation to deliver wheat – i.e., the use of the relation “owe”, and the use of a reified obligation such as OB-1 – would not work in this case. We will see why in the following section, in which we study this example in greater detail.

4. Definition and theory in jurisprudence

Recall the questions raised by Professor Grey and Professor Donahue in their contributions to the *Nomos* symposium on *Property* in 1977. Grey observed that the concept of property has tended to disintegrate in the modern era, as the Hohfeldian “bundle-of-rights” analysis gained primacy. Donahue acknowledged that the concept of property was in trouble, but also noted a countervailing tendency to agglomerate the Hohfeldian bundle of rights in a single legal person and a single

thing. In the previous section, we discovered a possible explanation for Donahue's agglomerative tendency. A contract claim is a pure Hohfeldian bundle of rights, as we have seen, but it does not disintegrate because there are *computational* advantages in treating it as a persistent object.

In this section, I will develop this analysis further, and apply it to the more complex example of corporate securities. But first, I will draw some connections between the computational model developed in the previous section and two additional strands of thought in the jurisprudential literature, one by H. L. A. Hart and one by Richard Posner.

The title of this section is borrowed from the title of Hart's inaugural lecture in the Chair of Jurisprudence at Oxford in 1953 (Hart 1984). In this lecture, Hart addresses several questions that had proven to be puzzling for the legal philosophers of his day: What is a right? What is a duty? What is a corporation? The tendency was to view these questions as a request for a traditional definition *per genus et differentiam*, and then to construct elaborate theories about the peculiar nature of the genus in these particular cases. Hart proposes, instead, a method of contextual definition. First, look at the term embedded in a complete sentence: "A has a right to be paid £10 by B", "Smith & Co. Ltd., a corporation, has a contract with B", etc. Then specify the conditions under which such sentences would be true, and show how the sentences in question could be used to draw legal conclusions. If we follow this method, Hart says, the philosophical puzzles will disappear. Here, for example, are his remarks on the concept of a corporation:

I have argued that if we characterize adequately the distinctive manner in which expressions for corporate bodies are used in a legal system, then there is no residual question of the form 'What is a corporation?'. There only *seems* to be one if we insist on a form of definition or elucidation which is inappropriate. Theories of the traditional form can only give a distorted account of the meaning of expressions for corporate bodies because they all, in spite of their mutual hostility, make the common assumption that these expressions must stand for or describe something, and then give separate and incompatible accounts of its peculiarity as a complex or recondite or a fictitious entity; whereas the peculiarity lies not here but in the distinctive characteristics of expressions used in the enunciation and application of rules. (Hart 1984, p. 42)

Although Hart was working within the tradition of linguistic philosophy prevalent at Oxford in the 1940s and 1950s, the reader should recognize that we have used essentially the same method of contextual definition in our computational analysis of Brubaker's claim. At the beginning of the previous section, we looked at examples of two traditional forms of definition, one in which a `SmallBusinessCorporation` was defined as a special case of the genus `Corporation`, and another in which the relationship of `Control` between an `Actor` and a `Corporation` was defined in terms of the preexisting relations `Own` and `Issued`. But when we constructed a model of Brubaker's claim for the delivery of

the wheat from Armstrong, we did not resort to either of these modes of definition. Instead, we treated the claim C-34 as a primitive object that would always be embedded inside an ownership relation, and we stipulated that this ownership relation would always be embedded inside a particular system of deontic modalities over actions.¹¹ In Hart's terms, what we did was to characterize "the distinctive manner in which expressions for [claims] are used in a legal system", and once we did this, there was no further need to answer the question: What is a claim? Hart could not have anticipated the technical details of a representation language such as *LLD*, of course, but he probably would have approved our use of contextual definitions.

When Hart used the word "theory" in the title of his inaugural lecture, he was referring pejoratively to the "vast and irreconcilable theories" (Hart 1984, p. 23) that had arisen to answer the simple questions: What is a right? What is a corporation? These theories are superfluous, Hart says, once we have arrived at a proper understanding of contextual definitions. But does this mean that we have no need at all for "theory" in our jurisprudence? Surely, the answer is: No.¹² In fact, in order to complete our analysis of the ownership relation and extend it to the case of corporate securities, we will find it useful to adopt a body of theoretical work that was first introduced into the jurisprudential literature by Richard Posner.

Recall the main line of argument in the previous section of this article: Assume that we want to assert that an ordinary contract claim is freely assignable. There are several ways to do this, but the best way, computationally, is to create a new object and a new ownership relation that carries with it the Hohfeldian power to transfer. We thus have a partial explanation for the emergence of abstract property rights, divorced from concrete material objects.

However, this is only a partial explanation. It does not explain why we would want the contract claim to be freely assignable in the first place. The rest of the explanation is given by economic theory, and it is now a conventional part of the literature on law and economics (Demsetz 1967; Posner 1973; Ackerman 1975). The transferability of property rights insures that resources can be shifted from less productive uses to more productive uses through voluntary exchange, thus increasing "economic efficiency" overall.¹³ This is true for concrete physical objects, such as land, buildings, or machines; it is also true for contract claims. For example, Cadbury might discover on August 15th that he could make a substantial profit in his mill if he had a guaranteed source of 5,000 bushels of wheat on September 1st, whereas Brubaker might discover on the same date that his mill was already running behind schedule and beyond capacity. Thus the contract claim would be worth more to Cadbury than to Brubaker, and it would be rational for Brubaker to sell it. Of course, Cadbury could always enter into a separate contract with Brubaker for the purchase of the wheat itself, as originally required under English law, but this is a cumbersome solution. It is much easier to assign the contract claim itself.

Equally important is another insight from economic theory. It is easy to see that the contract claim – reified into a piece of "property" that is "owned" by Brubaker

– acquires economic characteristics that are systematically related to Armstrong and his wheat. If the summer crop is poor, the value of the contract claim rises; if Armstrong becomes insolvent, the value of the contract claim falls; as the delivery date approaches, the value of the contract claim becomes more determinate. In principle, we can derive these characteristics directly from our representation of C-34 as an object embedded in a Hohfeldian “bundle-of-rights”. To extend these ideas to the case of corporate securities, however, we need to look at these derivations from a slightly different perspective. Suppose our goal is to design a system of Hohfeldian rights that enables a business organization to raise capital from a large and diffuse group of investors, by guaranteeing them a share of future profits. Some of the design constraints are discussed by Posner in the following passage:

The corporate form is the normal solution that the law and business practice have evolved to meet these problems The shareholder (as the investor in a corporation is known) is protected against being exploited by the entrepreneur or promoter by two methods. The first consists of a complex of legal rights vis-à-vis management and any controlling group of shareholders, such as the right to cast votes for candidates to the board of directors (which oversees management) in proportion to the number of shares owned by the shareholder. The second protection lies in the fact that equity interests in a corporation are broken up into shares of relatively small value which can be, and in the case of most of the larger corporations are, traded in organized markets. The shareholder need not make a large investment and if he wants to liquidate his investment by selling his shares he can do so quickly and cheaply. (Posner 1973, p. 177)

With these protections (and others) in place, the shareholder can view his investment as the economic equivalent of a future stream of income from the corporation, so that its value will rise or fall with the success or failure of the underlying business.

We now have the background we need to construct a computational representation of the rights of stockholders and bondholders: (1) We will use Hart’s method of contextual definition, revised and updated to reflect our current computational model. This means that we will treat *Stocks* and *Bonds* as simple mass terms, which can be owned and transferred, in whole or in part, and we will embed these ownership relations and transfer events inside a system of deontic modalities that represents a bundle of rights, in the sense of Hohfeld. (2) The particular system of Hohfeldian rights will be determined by economic theory, as suggested by Posner. For example, an owner of common stock will be subject to the *liability* of receiving periodic distributions of income from the corporation, and will have the power, by virtue of the voting rights attached to the stock, to modify the corporation’s dividend policies whenever this seems beneficial. On the other hand, a stockholder’s *claim* to corporate assets in a liquidation will be given a very low priority. By contrast, the owner of a bond will have a higher priority over the corporate assets in the event of a liquidation, and will have an absolute claim – not just an expectation –

to receive periodic distributions of income. The economic effect of this Hohfeldian “bundle-of-rights” will be to ensure that a stock has a relatively high rate of return and a relatively high level of risk, as compared to a bond.

To see how to carry out this construction, let us look in greater detail at the rights of a stockholder and a bondholder to the periodic distribution of income. For a particular corporation named Smith&Co and for a particular class of stock named Stock19, we might write out the deontic rule shown in Figure 8. This is a free choice permission for the distribution of cash with respect to the security Stock19. Although we cannot say precisely what this rule does until we have analyzed the definition of the action *Distribute*, the intended interpretation is as follows: The total amount of cash distributed is $N1$ dollars; the recipients of the cash are the owners of the shares of Stock19; the ratio of the amount of cash distributed to the number of shares of stock issued is $R1$. Since this is a free choice permission, in order to carry out the action, the corporation must choose the quantity $N1$ subject to the constraints $N1 = R1 * D1$ and $N1 \leq E1$, where $D1$ is the total number of shares issued and $E1$ is the surplus available for distribution. Alternatively, and equivalently, the corporation can choose the quantity $R1$, but subject to the same constraints. In practice, this is the way the distribution is usually described. For example, we might say that “the dividend is \$3.50 per share”.

In defining the action *Distribute*, we would like to generalize beyond the particular corporation named Smith&Co and the particular class of stock named Stock19, so we replace these constants with variables¹⁴ and write the left-hand side of our definition as shown in Figure 9. The right-hand side of our definition will have two parts. The first part *decrements* the amount of cash owned by the corporation, and is relatively straightforward. See Figure 10. Not surprisingly, this part of the definition only makes use of the number N that represents the total amount of cash involved in the distribution.

The second part of the definition is more complex. The problem here is to *increment* the amount of cash owned by the stockholders, but in precisely the right proportions, and to do this we need a universally quantified implication¹⁵ as shown in Figure 11. This part of the definition makes use of the ratio R and the mass term $?Security$, both of which are variables. (Within the free choice permission PE-5, of course, the variable $?Security$ would be bound to Stock19.) Intuitively, the implication says that, to every Actor A who owns D units of the specified $?Security$, whatever that security might be, the Corporation C transfers $M = R * D$ dollars in Cash. Thus, if a particular stockholder is known to exist, that stockholder would receive an increment of cash by the operation of this rule. But the rule has a *universal* and *hypothetical* import, even in the absence of a known stockholder. We can use it, for example, to ask “What if?” questions about the owners of arbitrary quantities of Stock19. This is an important aspect of the concept of a distribution of cash “with respect to a class of stock,” and it is therefore an important part of our contextual definition of stock ownership.

```

(Permit 'PE-5
 {condition
  (Issued -
   {subject (Corporation 'Smith&Co)}
   {object (Stock19 -
            {objectof
             (Measure -
              {unit      (Share 'S-1)}
              {quantity (Number D1)}}))})
  AND
  (Surplus -
   {subject (Corporation 'Smith&Co)}
   {object (Number E1)})
  AND ''N1 = R1 * D1'' AND ''N1 <= E1''}
 {action
  (Distribute -
   {agent (Corporation 'Smith&Co)}
   {object (Cash -
            {objectof
             (Measure -
              {unit      (Dollar 'D-1)}
              {quantity (Number N1)}}))})
   {ratio (Number R1)}
   {class (Stock19 -)}
   {time1 (Time T1)}
   {time2 (Time T2)}}})

```

Figure 8. Permission for the distribution of cash by a corporation.

```

(Distribute -
 {agent (Corporation C)}
 {object (Cash -
          {objectof
           (Measure -
            {unit      (Dollar 'D-1)}
            {quantity (Number N)}}))})
 {ratio (Number R)}
 {class (?Security -)}
 {time1 (Time T1)}
 {time2 (Time T2)}}

<==

```

Figure 9. Defining the action Distribute.

```

(Action -
  {agent (Corporation C)}
  {event
    (StateChange -
      {relation1
        (Own -
          {subject (Corporation C)}
          {object (Cash -
            {objectof
              (Measure -
                {unit (Dollar 'D-1')}
                {quantity (Number J1)}})}})}
        {relation2
          (Own -
            {subject (Corporation C)}
            {object (Cash -
              {objectof
                (Measure -
                  {unit (Dollar 'D-1')}
                  {quantity (Number J2)}})}})}}}
        AND 'J2 = J1 - N'}
      {time1 (Time T1)}
      {time2 (Time T2)}})

```

Figure 10. Decrementing the amount of cash owned by the corporation.

As a further check on the adequacy of our formalization, let us now see how to represent the analogous rights of a bondholder to the periodic distribution of cash.¹⁶ It turns out, happily, that our definition of the action *Distribute* remains the same, and the only part of the formalization that needs to be changed is the deontic modality. Assume that the corporation *Smith&Co* has also issued a security named *Bond23*. Instead of a free choice permission, we now need a conditional obligation, as shown in Figure 12. In this example, the interest rate *R2* is assumed to be an attribute of *Bond23*, and it is included as part of the condition on the obligation. This means that the quantity of cash to be distributed, *N2*, is fully determined. The reader should be able to verify that our previous definition of *Distribute* correctly characterizes the actions that *Smith&Co* is obligated to take with respect to its bondholders. We have thus represented an important part of the contextual definition of bond ownership.

When we analyzed the assignability of a simple claim for the delivery of wheat, in the previous section of this article, we noted that it was not absolutely necessary to create an abstract object, *C-34*, to serve as the carrier of *Armstrong's* obligation. There were alternative solutions that might have worked just as well. For example, we could have used a representation of the relationship among physical persons and concrete objects denoted by the English word "owe". But now that we have seen how to represent the rights of stockholders and bondholders using abstract

```

For all Actors A:

(Action -
  {agent (Corporation C)}
  {event
    (StateChange -
      {relation1
        (Own -
          {subject (Actor A)}
          {object (Cash -
            {objectof
              (Measure -
                {unit (Dollar 'D-1')}
                {quantity (Number K1)}})}}))
        {relation2
          (Own -
            {subject (Actor A)}
            {object (Cash -
              {objectof
                (Measure -
                  {unit (Dollar 'D-1')}
                  {quantity (Number K2)}})}}))
          AND 'K2 = K1 + M'}}
        {time1 (Time T1)}
        {time2 (Time T2)}}
      <==
      (Own -
        {subject (Actor A)}
        {object (?Security -
          {objectof
            (Measure -
              {unit (Unit -)}
              {quantity (Number D)}})}}))
    )
  )

```

Figure 11. Incrementing the amount of cash owned by the stockholders.

mass terms for Stocks and Bonds, it should be clear that these alternative solutions would not work in this case. A relationship here among physical persons and concrete objects would have to encompass hundreds or thousands of entities, many of which would be unknown or even hypothetical. This would not be feasible. Nor would it be feasible to work with reified deontic rules, as we did with the obligation OB-1. The rules stating primary obligations among the physical persons involved in the corporate enterprise would be mutually dependent, and the interactions would be exponential in number. Finally, as we have seen in our brief discussion of economic theory, it is important that the financial characteristics of an investment in the company have a simple relationship to the success or failure of the underlying business, and this relationship would be obscured if it had to be stated in terms of

```

(Oblige 'OB-7
  {condition
    (Issued -
      {subject (Corporation 'Smith&Co)}
      {object (Bond23 -
        {objectof
          (Measure -
            {unit (Dollar 'D-1)}
            {quantity (Number D2)}}))})
    AND
    (InterestRate -
      {subject (Bond23 -)}
      {object (Number R2)})
    AND 'N2 = R2 * D2'
  {action
    (Distribute -
      {agent (Corporation 'Smith&Co)}
      {object (Cash -
        {objectof
          (Measure -
            {unit (Dollar 'D-1)}
            {quantity (Number N2)}}))})
      {ratio (Number R2)}
      {class (Bond23 -)}
      {time1 (Time T1)}
      {time2 (Time T2)}})

```

Figure 12. Obligation for a corporation to distribute cash to its bondholders.

primary deontic rules. For all of these reasons, the creation of abstract objects like “stocks” and “bonds” that can be “owned” and “transferred” was an essential step in the development of corporate law.

It is interesting to note that H. L. A. Hart made a similar point in his lecture on *Definition and Theory in Jurisprudence*. Hohfeld had sketched out an analysis of the rights of corporate stockholders in one of his early articles (Hohfeld 1909), using his fundamental legal conceptions, of course, and Hart responded with both praise and criticism in the following passage:

It is easy to see that a statement about the rights of a limited company is not equivalent to the statement that its members have those same rights. A conveyance by Smith & Co. Ltd. to the sole shareholder Smith is of course not a conveyance by Smith to Smith. But a few theorists, among them Hohfeld, have stated this type of theory with a requisite degree of subtlety. Hohfeld saw that to say that Smith & Co. Ltd. has a contract with Y was, of course, not to say the same thing about the members of the company: he thought it was to say something different and very complicated about the way in which the capacities, rights, powers, privileges, and liabilities of the natural persons

concerned in the company had been affected. Though more formidable in this guise, the theory fails because, although it gives us the legal consequences upon the individuals of the original statement, it does not give us the force and meaning of that statement itself. The alleged paraphrase is less than the original statement ‘Smith & Co. Ltd. has a contract with Y’ because it gives no hint of what the original statement is used to do, namely, to draw a conclusion of law from special rules relating to companies and from rules extended by analogy from the case of individuals. (Hart 1984, p. 41)

Although Hart could not have been thinking in computational terms, as we might be today, he was clearly thinking about the inferences that lawyers make from sentences that include the terms “corporation”, “stock”, “bond”, etc. It might be possible, in principle, to eliminate these terms and translate these sentences into an equivalent bundle of Hohfeldian rights, Hart says, but such a translation would obscure the way lawyers use sentences of this sort to draw legal conclusions.

Today we can add a new insight to these observations: A translation that eliminated abstract objects like “stocks” and “bonds” (and “corporations”) could not be used at all to draw legal conclusions, because the necessary inferences would be too complex.

5. Conclusion

One of the main goals of jurisprudence is to clarify. Wesley Hohfeld’s fundamental legal conceptions certainly helped to clarify our thinking about property rights at the beginning of the 20th century. But the success of Hohfeld’s analytical method led to another puzzle. Why don’t abstract objects like debts and stocks and bonds just dissolve into their associated “bundle-of-rights”? Why, when the subject is corporate securities, do we insist on talking about “things” that are “owned” by “persons”? In this article, I have attempted to answer these questions by analyzing a computational model that was developed, over a number of years, as part of my work on AI and Law. Hopefully, the computational model has added some clarity to the subject.

Methodology tends to evolve in jurisprudence, often with an impetus from outside the field. Beginning in the 1950s, H. L. A. Hart made significant contributions to our understanding of law by applying some of the methods of ordinary language analysis that had been pioneered by J. L. Austin at Oxford and Ludwig Wittgenstein at Cambridge, among others. Twenty-five years later, Richard Posner took the field in a different direction by systematically applying economic theory to issues that had not previously been seen as economic. In the final section of this article, I have used the economic analysis of Posner to specify the Hohfeldian bundle of rights that must be attached to a particular corporate security, but for the detailed structure of these rights, I have replaced the linguistic philosophy of Hart with a more modern computational equivalent.

I believe that this methodology shows promise for future work. It is a hybrid methodology, part economic theory and part computational theory. It would be interesting to see if it could be applied elsewhere in the law.

Acknowledgements

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Notes

¹ In writing this abbreviated history, I have relied upon: Honoré (1961), Smith (1976), Donahue (1980), Grey (1980), Munzer (1990).

² Commentators on Hohfeld's system have often proposed minor variations in terminology, and I have adopted some of these variations here: (1) Hohfeld used the words "right" and "no-right" in place of the words "claim" and "no-claim" in Figure 1, since he considered the correlative of a duty to be a "right" in the strict sense. However, he also suggested that the word "claim" would be a reasonable synonym. (2) Hohfeld used the word "privilege" for the opposite of a duty, although he suggested "liberty" as a possible synonym. One problem with the word "privilege", as Hohfeld noted, is that it also connotes a special legal advantage. (3) Hohfeld used the word "disability" for the opposite of a power, but I have substituted the word "no-power" to emphasize the structural similarity to the concept of a "no-claim".

³ For a discussion of the relevance of such a distinction to the "takings clause" in the Fifth Amendment to the U.S. Constitution, see Ackerman (1977). I will not pursue this issue in the present article.

⁴ For a sharp critique of Grey's conclusions on this point, see Munzer (1990, pp. 31–36).

⁵ For a discussion of the problems involved in reasoning about such relationships, see McCarty and van der Meyden (1991).

⁶ For a discussion of the expressive power of this representation, and some of the techniques for reasoning about it, see McCarty and van der Meyden (1992).

⁷ Formally, this modality is defined using a Grand Permitted Set, exactly as in McCarty (1983). The only difference is that the subworlds in the Grand Permitted Set are now interpreted as actions that the agent is able to do, and the agent's actual actions are constrained to lie within this set.

⁸ The dual of $\mathbf{E}\langle\phi \mid \alpha\rangle$ is also interesting. Define $\mathbf{C}\langle\phi \mid \alpha\rangle \equiv \neg\mathbf{E}\langle\phi \mid \neg\alpha\rangle$ and interpret it as a *causal* modality. This modality is useful in a number of contexts, but will not be used in the present article.

⁹ There is a substantial literature on the proper way to formalize the correlation between Hohfeldian claims and Hohfeldian duties, and several authors have argued that the simple formalization adopted in this article is problematical in certain situations. See, e.g., Herrestad and Krogh (1995). However, the main thesis of the present article does not turn on the subtle distinctions made by these authors. The reader may substitute here any of the alternative formalizations in the literature, and verify that the main points about the concept of ownership remain valid.

¹⁰ In a previous paper (Schlobohm and McCarty 1989), I represented a legal power as a weak permission, so that X would have the power to bring about a change in the relation R just in case X 's action to change R was not forbidden. Although this purely deontic representation seems adequate for encoding the powers of a trustee, which was the primary application in Schlobohm and McCarty (1989), Layman Allen has pointed out to me that it does not correspond to Hohfeld's original conception of a power. I have therefore replaced the modality: $\neg\mathbf{F}$ with the structurally similar modality \mathbf{E} . This new representation allows us to say that X has a power over Y with respect to ΔR , but that X is nevertheless forbidden to exercise that power. Layman Allen's position on this issue is also followed, up to a point, in Jones and Sergot (1996). An even finer grained representation would distinguish between physical acts and legal acts, and encode a power as the ability to perform a particular physical act (e.g., reciting a sentence, signing a document) which is then causally linked to a change in a legal relationship. Compare Jones and Sergot (1996). However, for present purposes, this finer grain does not seem to be necessary.

¹¹ Another way to put this point is to say that a claim is *syntactically simple* but *semantically complex*.

¹² "Hence, though theory is to be welcomed, the growth of theory on the back of definition is not" (Hart 1984, p. 25).

¹³ I will not discuss here the various elaborations and qualifications of this principle that are necessary to apply it to even mildly realistic situations. See, e.g., Michelman (1982) and Demsetz (1982).

¹⁴ Notice that we have introduced a new notation here for the *variable sort*, $?Security$. Although this is a minor technical detail, it is convenient to assume that an individual symbol is a variable unless marked otherwise, while a sort symbol is a constant unless marked otherwise.

¹⁵ When the body of a definition includes a universally quantified implication, the proper logical interpretation of the language is *intuitionistic*, not classical. Such definitions are studied in (McCarty 1993).

¹⁶ For simplicity, the periodic nature of the distribution has been omitted here from our representation of the rights of both stockholders and bondholders. Typically, dividend and interest payments are made quarterly, and this condition can be added to the permission PE-5 and the obligation OB-7 by stating that the Time T_1 occurs at the beginning of the quarter.

References

- Ackerman, B. A. ed. (1975). *Economic Foundations of Property Law*. Little, Brown and Co.
- Ackerman, B. A. (1977). *Private Property and the Constitution*. Yale University Press.
- Ames, J. B. (1913). *The Disseisin of Chattels*. In *Lectures in Legal History*, 172–191. Harvard University Press.
- Blackstone, W. (1791). *Commentaries on the Laws of England*. 11th edition. London.
- Brachman, R. J., Fikes, R. E., and Levesque, H. J. (1985). KRYPTON: A Functional Approach to Knowledge Representation. In Brachman, R. J. and Levesque, H. J., (eds.) *Readings in Knowledge Representation*, 411–429. Morgan Kaufmann.
- Demsetz, H. (1967). *Toward a Theory of Property Rights*. *American Economic Review, Papers and Proceedings* 57: 347–359.
- Demsetz, H. (1982). *Professor Michelman's Unnecessary and Futile Search for the Philosopher's Touchstone*. In Pennock, J. R. and Chapman, J. W. (eds.) *Nomos XXIV: Ethics, Economics, and the Law*. New York University Press.
- Donahue, Jr. C. (1980). *The Future of the Concept of Property Predicted from its Past*. In Pennock, J. R. and Chapman, J. W. (eds.) *Nomos XXII: Property*, 28–68. New York University Press. Papers presented at a meeting of the American Society for Political and Legal Philosophy, 1977.

- Grey, T. C. (1980). The Disintegration of Property. In Pennock, J. R. and Chapman, J. W. (eds.) *Nomos XXII: Property*, 69–85. New York University Press. Papers presented at a meeting of the American Society for Political and Legal Philosophy, 1977.
- Hart, H. L. A. (1984). Definition and Theory in Jurisprudence. In *Essays in Jurisprudence and Philosophy*, 21–48. Oxford University Press.
- Herrestad, H. and Krogh, C. (1995). Obligations Directed from Bearers to Counterparties. In *Proceedings of the Fifth International Conference on Artificial Intelligence and Law*, 210–218. ACM Press.
- Hohfeld, W. H. (1909). The Nature of Stockholders' Individual Liability for Corporation Debt. *Columbia Law Review* 9: 285.
- Hohfeld, W. H. (1913). Some Fundamental Legal Conceptions as Applied in Judicial Reasoning. *Yale Law Journal* 23: 16.
- Hohfeld, W. H. (1917). Fundamental Legal Conceptions as Applied in Judicial Reasoning. *Yale Law Journal* 26: 710.
- Hohfeld, W. H. (1964). *Fundamental Legal Conceptions*, vii–xv. Yale University Press. Foreword by Arthur L. Corbin.
- Honoré, A. M. (1961). Ownership. In Guest, A. G. (ed.), *Oxford Essays in Jurisprudence*, 107–147. Oxford University Press.
- Jones, A. J. I. and Sergot, M. (1996). A Formal Characterization of Institutionalized Power. *Journal of the IGPL* 4: 429–445.
- Lindahl, L. (1977). *Position and Change: A Study in Law and Logic*. D. Reidel.
- McCarty, L. T. and Sridharan, N. S. (1981). The Representation of an Evolving System of Legal Concepts: II. Prototypes and Deformations. In *Proceedings of the Seventh International Joint Conference on Artificial Intelligence*, 246–253.
- McCarty, L. T. and van der Meyden, R. (1991). Indefinite Reasoning with Definite Rules. In *Proceedings of the Twelfth International Joint Conference on Artificial Intelligence*, 890–896.
- McCarty, L. T. and van der Meyden, R. (1992). Reasoning about Indefinite Actions. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Third International Conference (KR92)*, 59–70. Morgan Kaufmann.
- McCarty, L. T. (1977). Reflections on TAXMAN: An Experiment in Artificial Intelligence and Legal Reasoning. *Harvard Law Review* 90: 837–893.
- McCarty, L. T. (1980). The TAXMAN project: Towards a Cognitive Theory of Legal Argument. In Niblett, B. (ed.) *Computer Science and Law: An Advanced Course*, 23–43. Cambridge University Press.
- McCarty, L. T. (1983). Permissions and Obligations. In *Proceedings of the Eighth International Joint Conference on Artificial Intelligence*, 287–294.
- McCarty, L. T. (1986). Permissions and Obligations: An Informal Introduction. In Martino, A. A. and Socci Natali, F. (eds.) *Automated Analysis of Legal Texts: Logic, Informatics, Law*, 307–337. Elsevier North-Holland. Also available as Rutgers Technical Report LRP-TR-19.
- McCarty, L. T. (1989). A Language for Legal Discourse. I. Basic Features. In *Proceedings of the Second International Conference on Artificial Intelligence and Law*, 180–189. ACM Press.
- McCarty, L. T. (1993). Circumscribing Embedded Implications (without Stratifications). *Journal of Logic Programming* 17: 323–364.
- McCarty, L. T. (1995). An Implementation of *Eisner v. Macomber*. In *Proceedings of the Fifth International Conference on Artificial Intelligence and Law*, 276–286. ACM Press.
- Michelman, F. I. (1982). Ethics, Economics, and the Law of Property. In Pennock, J. R. and Chapman, J. W. (eds.) *Nomos XXIV: Ethics, Economics, and the Law*. New York University Press.
- Munzer, S. R. (1990). *A Theory of Property*. Cambridge University Press.
- Posner, R. A. (1973). *Economic Analysis of Law*. Little, Brown and Co.

- Schlobohm, D. A. and McCarty, L.T. (1989). EPS-II: Estate Planning with Prototypes. In Proceedings of the Second International Conference on Artificial Intelligence and Law, 1–10. ACM Press.
- Smith, J. C. (1976). Legal Obligation. The Athlone Press, University of London.

