



A general equilibrium model of crime and punishment [☆]

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Received 4 August 1997; received in revised form 9 December 1998

Abstract

A model of crime and punishment is developed where individuals who differ in their earnings abilities choose between work and crime, taking the probability and consequences of punishment into account. An aggregate relationship between the probability of punishment and the level of crime is derived. There is also a relationship between enforcement spending, the number of criminals and the number punished. In such an economy, the possibility of multiple equilibria and the effects of changes in enforcement spending and in inequality on the levels of crime and punishment are discussed; there is also discussion of social welfare and voting behavior. ©1999 Elsevier Science B.V. All rights reserved.

JEL classification: D72; K42

Keywords: Crime; Multiple equilibria; Inequality

1. Introduction

Crime is an important social and political problem. The costs it imposes on victims, the public purse, the economy and society can be considerable, but there is little agreement amongst policy makers (and others) on what should be done about it. Crime is, undoubtedly, a complex phenomenon, and many disciplines have a role to play in helping us understand it. This paper is written in the belief that an economic approach has something to offer.

In Becker's (Becker, 1968) seminal paper an agent decides whether to commit crime, and how much crime to commit, by comparing the benefits and costs of crime with those of alternative activities. So the probability and magnitude of punishment should affect the

[☆] Versions of the paper were presented in seminars at the Pennsylvania State University and the Universities of Birmingham and Lancaster.

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level of crime, as should the proceeds of criminal activity and the return to work (envisaged as an alternative to crime). Most of the economic literature on crime¹ follows the Beckerian tradition, and this paper is no exception. However, it moves from the analysis of a single agent to determine the total level of crime in a general equilibrium setting. If all agents are the same, then the problem is trivial; however, such an assumption (of a representative agent), whilst common in economics, is particularly inappropriate for analyzing crime; one of the salient facts about crime is that it is committed by a minority of the population and that some individuals commit crimes in circumstances when others do not. An aggregate model of crime thus requires an assumption that agents are heterogeneous; this is the approach we take; we assume that agents differ in their earning abilities, so have differing incentives to participate in crime, the alternative to work in the model. Using this tractable yet not unreasonable way of modelling agent heterogeneity, we construct a model of the determination of the aggregate level of crime with plausible microfoundations. This enables us to analyze the effects of changes in a number of factors, including policy variables, on the level of crime.

Our model comprises two basic relationships. The first relates the probability of punishment to the level of crime. The second connects the probability of punishment to the level of crime and resources spent on enforcement. The equilibrium level of crime and the probability of punishment are then determined simultaneously.

As part of the economy-wide coordination problem we consider the existence of multiple equilibria, a possibility that explains why levels of crime can differ significantly between regions with similar characteristics.² This idea has appeared occasionally in the literature without a fully satisfactory and rigorous argument in the context of a formal model having been developed.³ The intuition is as follows: if crime is high, then, with given resources spent on enforcement, the probability of punishment is low, hence implying a high level of crime. Alternatively, a low level of crime can be an equilibrium as well – if crime is low, the probability of punishment is high, so little crime is committed.

A number of comparative statics results are derived (assuming a stable equilibrium); for example, provided crime is initially not too high, an increase in inequality will raise crime, a theoretical result with empirical support (see Ehrlich, 1973; Freeman, 1996).

Some extensions to the basic model are considered; for example, we suppose that crime reduces the attractiveness of legitimate employment and that taxes to pay for enforcement expenditures are paid by the employed, and show how this can affect our results. Optimal and endogenous policy are also considered in the model.

The paper is organized as follows: Section 2 presents the basic model. The possibility of multiple equilibria is investigated in Section 3 and the following Section 4 presents some comparative statics results. Extensions to the basic model are discussed in Section 5 whereas Section 6 explores policy questions and there follows a final, concluding section.

¹ Recent surveys of the literature on the economics of crime are Fajnzylber et al. (1997) and Eide (1997).

² For evidence on this see, for example, Glaeser et al. (1996), p. 508.

³ The papers I am aware of which discuss multiple equilibria in the context of crime are by Neher (1978), Sah (1991), and Glaeser et al. (1996).

2. The basic model

We assume an economy with a population of n members; of these $n - m$ are ‘incorruptible’ and never commit crime.⁴ The remainder, m , become criminals under certain circumstances; n , $n - m$ and m are all large. The only decision a (corruptible) agent makes is whether to become a criminal or not, he does this by comparing the costs and benefits of criminality with the alternative, work. Let $c_i = 1$ if agent i becomes a criminal, otherwise let c_i be zero. Then the economy-wide crime level (C) is defined by $C \equiv \sum c_i$ and the crime rate by $c \equiv C/n$. So $0 \leq c \leq m/n$.

If (corruptible) agent i works, he receives a wage of w_i (which is also his marginal product); w_i is uniformly distributed between $\underline{w} - \alpha$ and $\underline{w} + \alpha$, so α parameterizes the degree of (wage) inequality in the economy.⁵ Incorruptible agents all work and receive wage \underline{w} . If an agent becomes a criminal and is not punished, his return (value of the goods stolen less cost of stealing them) is u_1 ; if he is punished, which occurs with probability p (which he takes as given), his return (u_1 less cost of punishment) is $u_2 (< u_1)$. Only criminals run the risk of punishment (there are no Type II errors although, if p is strictly less than unity, there are Type I errors).

An individual becomes a criminal if his expected net gain from criminal activity is non-negative,⁶ or if

$$pu_2 + (1 - p)u_1 - w_i \geq 0. \tag{1}$$

If this condition is satisfied for some, but not all, agents, then there is a critical level of the wage (w^*) such that an individual who can obtain this wage is indifferent between crime and work; agents who can earn higher wages work, while those with lower earnings potential turn to crime.⁷ So w^* is defined by

$$pu_2 + (1 - p)u_1 = w^*. \tag{2}$$

We can relate w^* and C as follows. If the critical wage is w^* , this means (with the assumption that earning ability is distributed uniformly) a proportion of the (potentially criminal) population $\{w^* - \underline{w} + \alpha\} / 2\alpha$ are criminals, and hence the number of criminals (and level of crime) is given by

$$C = \left(\frac{m}{2\alpha}\right) \{w^* - \underline{w} + \alpha\} \tag{3}$$

⁴ The assumption that there are some agents who never become criminals, as well as being realistic, plays a crucial part in the model. In our formulation, the marginal return to crime is constant, so if all corruptible agents are criminals, there must be some non-criminals to produce the goods stolen.

⁵ It is possible to interpret w_i in other ways, for example, it could represent the return to home production of the i th agent.

⁶ We make the inconsequential assumption that an agent who is exactly indifferent between becoming and not becoming a criminal does turn to crime.

⁷ We assume just one dimension of heterogeneity, relating to potential earnings, amongst agents. Individuals could differ in numerous other ways (conscience, ability to commit crime, disutility of work, etc.) but introducing another dimension of heterogeneity would generate additional complexities which would outweigh any possible benefits.

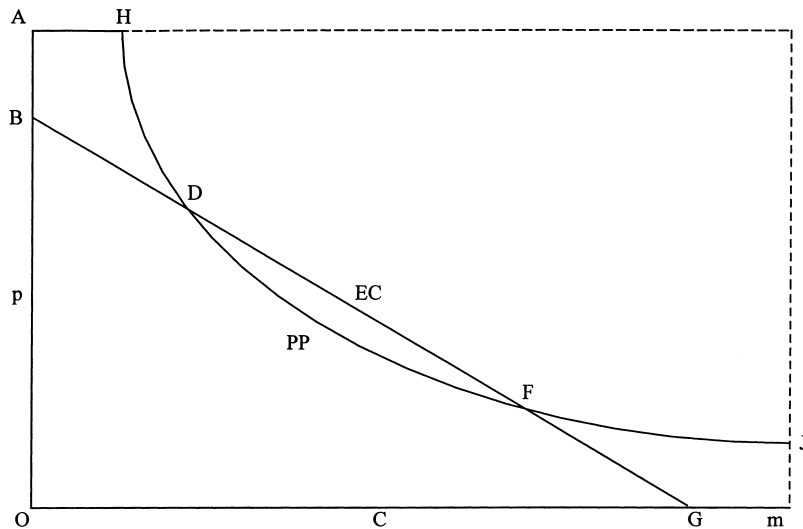


Fig. 1. The possibility of multiple equilibria.

By substituting from Eq. (3) for w^* into Eq. (2) we obtain:

$$pu_2 + (1 - p)u_1 = 2\alpha \frac{C}{m} + \underline{w} - \alpha, \quad (4)$$

or

$$C = \left(\frac{m}{2\alpha}\right) [pu_2 + (1 - p)u_1 - \underline{w} + \alpha] \equiv x. \quad (5)$$

Eq. (5) defines a relationship between the aggregate level of crime (C) and the probability of punishment (p), derived on the assumption that individuals optimize, taking the probability of punishment (if they become criminal) as given. We describe Eq. (5) as representing the 'Equilibrium Crime' or EC locus and portray it in Fig. 1. Its slope is $dp/dC = -2\alpha/m(u_1 - u_2)$, which is negative, as might be expected—a higher probability of punishment means that less crime is committed.⁸

In deriving Eq. (5) we have not imposed the constraint that C must lie between 0 and m . When this is done, the equation of the EC locus becomes

$$C = \max[0, \min\{x, m\}]. \quad (6)$$

Fig. 1 shows one possible EC locus, ABG. Crime is measured on the horizontal axis, whereas the vertical axis measures the probability of punishment. For values of p above B, where the EC locus cuts the vertical axis, crime is zero. When p falls to zero, crime is given by the distance OG. Of course, the EC locus may not intersect the vertical axis at a value of p

⁸ Our assumptions that the distribution of individuals' earnings' options is uniform and that the marginal return to crime is constant are crucial in generating a linear EC curve. If, instead, the distribution were normal, the curve would be downward sloping but convex. On the other hand, if crime were subject to diminishing marginal returns, this would tend to make the locus concave.

below unity. If this is so, some agents commit crime even if punishment is certain. It follows from Eq. (5) that if $p = 1$ then we have

$$C = \left(\frac{m}{2\alpha}\right)[u_2 - \underline{w} + \alpha], \tag{7}$$

so for crime to be positive if punishment is certain it is necessary (and sufficient) that $u_2 > \underline{w} - \alpha$.

It is also possible for the EC locus not to intersect the horizontal axis at a value of C less than m . Substituting $p = 0$ into Eq. (5), we obtain

$$C = \left(\frac{m}{2\alpha}\right)[u_1 - \underline{w} + \alpha], \tag{8}$$

which means that $u_1 < \underline{w} + \alpha$ is the condition for crime to be less than m even if the probability of punishment is zero (this states that the person with highest earning potential is better off working than committing crime even if he is certain not to be caught). If this inequality is reversed, then the EC locus intersects the $C = m$ locus at a positive value of p ; for lower values of p the EC locus then coincides with the $C = m$ locus.

Eq. (5) gives us one relationship between p and C . In order to complete the model, we need another relationship, which we now introduce. Let P be the number of criminals punished (so $p \equiv P/C$). Also, let E be total spending on law enforcement (police, law courts, prisons, etc.). We then postulate the following relationship (where subscripts denote partial derivatives):

$$P = \min[C, F(C, E)], \text{ with } F_1 \geq 0, F_2 > 0, F_{22} < 0. \tag{9}$$

Eq. (9) tells us that, provided not all the criminals are punished, more expenditure on law enforcement raises the number of criminals punished (for any given number of criminals). Also, for any given spending on law enforcement, the existence of more criminals means that more of them are punished (except in the limiting case); a rationale might be that if there are more criminals, it becomes easier to find and detect at least some for any given expenditure on enforcement. Eq. (9) is similar to Sah's Eq. (8) (Sah, 1991, p. 1278) and can be rewritten as

$$p = \min\left[1, \frac{F(C, E)}{C}\right], \tag{10}$$

which gives a second relationship between p and C . We consider two special cases of Eq. (10). First, when the number of criminals is irrelevant to the number of criminals punished (except as an upper bound), given E (i.e. $F_1 = 0$), we have

$$p = \min\left[1, \frac{G(E)}{C}\right], \tag{11}$$

where $G(E)$ now gives the number of criminals punished when expenditure on law enforcement is E . The other special case is where the level of expenditure on law enforcement determines the probability of punishment, so we have

$$P = \min[1, H(E)], \text{ where } H(E) \geq 0, H'(E) \geq 0. \tag{12}$$

We regard Eq. (11) as a more reasonable special case than Eq. (12) and shall use it sometimes in this paper. It seems very implausible that the level of expenditures on law enforcement determines the probability of punishment irrespective of the level of crime, which is what Eq. (12) states.⁹ For example, it surely requires fewer resources to generate any given probability of punishment if there are only ten criminals in the population than if there are ten million.

One particular functional form (which encompasses both special cases) that (9) might take is

$$P = \min[C, C^\beta E^\gamma], \quad \text{with } 0 \leq \beta \leq 1, \gamma > 0. \quad (13)$$

So if $\beta=0$, we have (11) with $G(E)=E^\gamma$, and $\beta=1$ gives (12) with $H(E)=E^\gamma$. If the coefficients β and γ sum to unity, there are constant returns to scale in enforcement technology-if resources spent on enforcement and the number of criminals double, twice as many criminals are punished as well. We believe it plausible to rule out decreasing returns-it is difficult to see how P could increase by less than two-fold if both E and C double, but it could well increase by more than twice if a higher density of criminals makes them easier to apprehend. (Note that we do not exclude γ being greater than unity.)

A perceptive reader may have noted that we have not discussed the effects of the level of crime on agents' decisions, nor specified how enforcement expenditure is financed. Since the only choice an agent makes is whether to become a criminal or not, our approach will be valid if crime affects criminals and non-criminals equally, and enforcement expenditures are financed in a way which does not affect the agent's decision whether to become a criminal. (We relax these somewhat implausible assumptions later.)

Our basic model has now been specified; the next task is to investigate the possibility of multiple equilibria.

3. Multiple equilibria

Multiple equilibrium levels of crime provide an explanation for large unexplained variances in crime rates between regions and economies with similar features. We consider four questions concerning such equilibria. (1) Can it be shown that multiple equilibria may exist? (2) What are the necessary conditions for multiple equilibria to exist? Failure of such conditions to hold would then rule out multiple equilibria. The best known necessary condition is the 'strategic complementarity' condition of Cooper and John (1988); however, this is a fairly weak condition, requiring the reaction function to be upward sloping (at least somewhere); a more stringent necessary condition is that the reaction function have a slope greater than unity at an equilibrium.¹⁰ We present a necessary condition for multiple equilibria in our model. (3) Are there multiple stable equilibria? Otherwise, the multiple

⁹ Nevertheless, a formulation similar to Eq. (12) is found in the literature on illegal immigration - that is, the proportion of illegal immigrants apprehended depends on the resources spent on enforcement (see Ethier, 1986, p. 58, for example).

¹⁰ A recent paper which uses this condition in an analysis of the possibility of multiple equilibria in transition economies is Fender and Laing (1998).

equilibrium result is much less interesting, as no more than one equilibrium will ever be observed (with positive probability). Of course, answering this question involves specifying the out-of-equilibrium behavior of the system. We do this, and argue that our system may well possess multiple stable equilibria. (4) Are the equilibria, as is often the case in multiple equilibria models, Pareto rankable? Perhaps surprisingly, the answer is negative.

Equilibrium is given by the values of p and C which satisfy both Eq. (5) and Eq. (10). Fig. 1 shows a possible setup; as we have drawn it, the EC locus (ABG) intersects the vertical axis at a value of p strictly less than unity and coincides with the vertical axis above B , until it reaches point A (where $p = 1$). Also, it intersects the horizontal axis before $C = m$ at point G ; if we define the level of crime at G as C_2 , then, (from Eq. (8)), $C_2 = m(u_1 - \underline{w} + \alpha)/2\alpha$.

We assume that the equation of the PP locus takes the special form Eq. (11), so AHJ is a typical locus. When $G(E) > C$, the locus coincides with the $p = 1$ line; this means that all criminals are punished, and, also, without any increase in spending on law enforcement, if more agents were to turn to crime, they too would be punished (except exactly at point H). Thereafter, the PP locus is a rectangular hyperbola until it cuts the vertical $C = m$ locus.

We first of all note that point A (where $p = 1, C = 0$) is an equilibrium. At this point no crime is committed; if a crime is committed, it is certain to lead to punishment, and this is enough to deter even the person for whom the opportunity cost of committing crime is lowest. In the case where the EC locus does not intersect the vertical axis at a value of less than unity, then we would have an equilibrium at the point where it intersects the AH segment of the PP locus, provided it does indeed intersect it. In this case we would have an equilibrium (with $p = 1, C > 0$); here, criminals would be certain of punishment, but this would not deter all potential criminals.

As we have drawn the curves, it seems there can be multiple equilibria—in Fig. 1, there are three equilibria; as well as the corner equilibrium, A , there are interior equilibria at D and F . We can demonstrate that there may indeed be multiple equilibria by giving an example; by combining Eq. (5) and Eq. (11) (for p less than unity) and manipulating, we obtain the following quadratic equation for C :

$$2\alpha C^2 - m\{u_1 - (\underline{w} - \alpha)\}C + m(u_1 - u_2)G(E) = 0, \tag{14}$$

which gives

$$C = \frac{m\{u_1 - (\underline{w} - \alpha)\} \pm m\sqrt{\{u_1 - (\underline{w} - \alpha)\}^2 - 8\alpha(u_1 - u_2)G(E)/m}}{4\alpha}. \tag{15}$$

If $u_1 = \underline{w}$ and $u_2 = \underline{w} - \lambda\alpha$, where λ is a constant the value of which will be specified in due course, Eq. (15) becomes

$$C = \frac{m}{4} \left[1 \pm \sqrt{1 - 8\lambda \frac{G(E)}{m}} \right]. \tag{16}$$

It is clear that we can, by appropriate choices of λ and of the functional form of $G(E)$, make the square root term in Eq. (16) take any value between 0 and 1. It follows that our model can admit two interior solutions for the crime rate. On the other hand, if $8\lambda G(E)/m > 1$, there are no interior equilibria. For example, suppose $\lambda = 1$ (which means that the EC locus intersects the vertical axis at precisely $p = 1$) and $G(E) = E$ (which means that $P = E$ as long as

$P \leq C$). Then from Eq. (16), the equilibrium crime rate becomes $(m/4)\{1 \pm \sqrt{(1 - 8E/m)}\}$. It follows that if $E < m/8$, there are two interior equilibrium crime rates; there will also be a ‘corner’ equilibrium with $C = 0, p = 1$. If $E = m/8$, then there is precisely one interior equilibrium; this is where the EC locus is tangential to the PP locus. There will also be a corner equilibrium in this case. For $E > m/8$, there are no interior equilibria - this corresponds to the case where the PP locus lies above the EC locus, except at the corner equilibrium, which continues to obtain. To calculate the corresponding equilibrium values of p , we note that under the above conditions the equation of the EC locus simplifies to $C = (m/2)(1 - p)$, substitute in the equilibrium values of C , and solve for p ; it may be checked that such values of p always lie between zero and unity.

So multiple equilibria may exist, and we now consider whether one or more of them is stable. For this purpose, we give an informal argument about what happens out of equilibrium. We assume that the system is always on the PP locus, so Eq. (10) always holds, but that there can be departures from the EC locus. Suppose that C , for a certain value of p , is above the level specified by Eq. (5), which means, the system is to the right of the EC locus, so some individuals are irrationally committing crime; that is, they are criminals even though Eq. (1) does not hold. An explanation might be that they do not use the true probability of punishment when making their decisions, but instead underestimate it. However, over time, they might be expected to revise upward their subjective probability of punishment in the light of experience (as suggested by Sah, 1991) and, accordingly, some of them cease being criminals and C falls. So, it seems reasonable to suppose that C falls to the right of the EC locus and, conversely, rises to its left, as agents adjust their subjective probability of being punished towards the ‘true’ probability. Given this assumption about the dynamics of the system, F and A are stable equilibria whereas D is unstable.¹¹ Summarizing:

Proposition 1. *For certain parameter values, there may be multiple stable equilibrium levels of crime.*

So far we have assumed that the equation of the PP locus takes the special form given in Eq. (11). What happens if we change this assumption? If we use Eq. (12), instead of Eq. (11), the PP locus is horizontal and there is a unique equilibrium. Intuitively, it is necessary for multiple equilibria to exist that the slope of the PP locus be less (at least somewhere) than the slope of the EC locus. From Eq. (10),

$$\left. \frac{dp}{dC} \right|_{PP} = \frac{1}{C} \left[F_1 - \frac{F}{C} \right]. \quad (17)$$

¹¹ We might ask whether any alternative dynamic specification would change this conclusion. One possibility is that there can be divergencies from the PP locus; perhaps the relationship given by Eq. (9) is a long-run relationship toward which the system adjusts over time-the system might be below the PP locus if there is slack in the enforcement relationship which means that not as many criminals are being punished as could be with existing resources, and conversely, we could be above the PP locus if resources in the enforcement sector are temporarily overstretched. In such circumstances, we would expect P , and hence p , to adjust in the direction of the PP locus. With such an assumption about the behavior of p , and retaining the original assumption about the adjustment of C , it turns out that points such as D , where the EC locus cuts the PP line from below, are saddlepoints, whereas points such as A and F are stable (sinks).

When the PP locus is given by Eq. (11), $F_1 = 0$, and the slope tends to minus infinity as C tends to zero; as the slope of the EC locus, $-\{2\alpha/(u_1 - u_2)m\}$, is negative and constant, the criterion is hence satisfied. When PP is given by Eq. (12), the slope is always zero, and so the necessary condition for multiple equilibria to exist is never satisfied.

We have also assumed that the EC locus cuts the PP locus at a value of C below m . However, it is quite possible that it cuts the $C = m$ locus above the PP locus. In this case, the point where the EC locus cuts the $C = m$ line would be a stable equilibrium, so there would then be two stable corner equilibria.

Finally, it turns out that multiple equilibria cannot be ranked in terms of the Pareto criterion. Workers are better off in a low-crime equilibrium, but, given that such equilibria tend to have higher probabilities of punishment, and this reduces the expected well-being of criminals, they are not Pareto superior. Zero-crime equilibria cannot be shown to be Pareto superior to equilibria with positive crime since it is possible that former criminals are worse off as workers in the zero-crime equilibrium.

4. Comparative statics results

It is straightforward to carry out comparative statics analysis, assuming a stable interior equilibrium. We first totally differentiate Eqs. (5) and (10), with $p < 1$:

$$\begin{pmatrix} 1 & \frac{2\alpha}{m(u_1 - u_2)} \\ C & (p - F_1) \end{pmatrix} \begin{pmatrix} dp \\ dC \end{pmatrix} = \begin{pmatrix} \frac{m-2C}{m(u_1 - u_2)} d\alpha \\ F_2 dE \end{pmatrix}. \tag{18}$$

Defining Δ as the determinant of the left-hand side matrix, $p - F_1 - 2C\alpha/m(u_1 - u_2)$, the comparative statics results can be written as follows:

$$\frac{dp}{d\alpha} = \frac{(m - 2C)(p - F_1)}{m(u_1 - u_2)\Delta}, \tag{19}$$

$$\frac{dp}{dE} = \frac{-2\alpha F_2}{m(u_1 - u_2)\Delta}, \tag{20}$$

$$\frac{dC}{d\alpha} = \frac{-C(m - 2C)}{m(u_1 - u_2)\Delta}, \tag{21}$$

$$\frac{dC}{dE} = \frac{F_2}{\Delta}. \tag{22}$$

In order to sign these expressions, we note that Δ is negative if and only if the slope of the PP locus is greater than that of the EC locus; our discussion in the last Section showed that this is a stability condition and we accordingly assume Δ negative. To sign Eq. (19), we observe that if the enforcement function takes the form $P = C^\beta E^\gamma$, as in Eq. (13) with $P < C$, then $p - F_1$ equals $(1 - \beta)p$ which is, provided β is strictly less than one, positive. Accordingly, we assume this to be the case, so that an increase in inequality (α) will raise crime and reduce the punishment rate, provided that C is initially less than $m/2$, that is, provided that less than half the (corruptible) population are criminals. The idea is that in these circumstances an increase in inequality reduces the wage income of the person who

was initially indifferent between crime and work, hence making crime individually more advantageous.

Proposition 2. *Provided that initially less than half the corruptible population are criminals, and the enforcement function takes the form $P = C^\beta E^\gamma$, with $\beta < 1$, then an increase in inequality, measured by α , will raise crime.*

According to Eqs. (20) and (22), an increase in enforcement spending raises the punishment rate and reduces the level of crime. These results are as expected. We would note, however, that the impact of an increase in enforcement spending on the crime rate depends not just on the parameters of the enforcement function itself, but on a number of other parameters as well, such as α ; the difference punishment makes to the well-being of criminals ($u_1 - u_2$) is relevant as well. If this is large, then an increase in enforcement spending may have a big effect on crime even if the direct effect ($-F_2$) is small. The intuition is that an increase in enforcement spending has the initial effect of raising both the level and probability of punishment; the effect on the level of crime depends on how the increase in p reduces the incentive to commit crime. As crime falls, then this means that p rises further, and so forth—so an increase in E can have a ‘multiplier’ effect on the level of crime (from Eq. (22), $1/\Delta$ might be interpreted as the ‘enforcement multiplier’). These effects can be illustrated quite easily diagrammatically, and the sizes of the various effects related to the slopes of the two curves. For example, an increase in enforcement spending shifts the PP locus upwards (more people are punished for any given level of crime); the shift is greater, the larger is F_2 . The extent to which this translates into a fall in crime obviously depends on the slopes of the two loci. The steeper the EC locus, the greater the impact of an increase in enforcement spending on crime will be, *ceteris paribus*. This is fairly obvious since the steeper the EC locus, the more sensitive crime is to an increase in p . Also, the impact will be greater, the steeper the PP curve. The idea is that the steeper this locus, the more a reduction in the number of criminals releases resources which can be devoted to apprehending the remaining lawbreakers.

The diagrammatic framework can also be used to illustrate an identification problem in the economic analysis of crime which, although recognized (e.g., Ehrlich, 1996, pp. 59–60), deserves to be mentioned. Suppose one is trying to estimate the effects of punishment probability on crime and one regresses the level of crime on the estimated punishment rate; suppose also that it is variations in the EC curve which are causing the crime rate to change, then in fact one is estimating the enforcement (PP) locus, which may be downward sloping, and this does not shed any light on whether a higher probability of punishment deters crime.¹²

It is also possible to derive results on the effects of a change in the average wage on crime; we do not present the algebra but it should be fairly clear that it means an inward shift in the EC locus, a fall in crime and a rise in the probability of punishment. (The explanation is obviously that if wages are higher, working is now more attractive compared with a life of crime.) However, we do not wish to stress this result, as our model does not incorporate two effects which might reverse the effect: firstly, if society is richer, and average wages are higher, the returns to crime may be higher (there is more to steal) and secondly, a higher

¹² Recent work has sought to overcome this problem by using appropriate instruments, as in Levitt (1996).

average wage (due to higher productivity) in the economy might make law enforcement more expensive (if productivity rises less in the law enforcement sector than on average in the economy), hence meaning that any given amount of expenditure on law enforcement reduces E in real terms.

5. Extensions

So far, we have not specified either how crime affects individuals, nor how the expenditure is financed; implicitly, we have assumed that crime affects both criminals and non-criminals equally, and enforcement spending is financed by a tax which is paid equally by criminals and workers, so that the choice whether to become a criminal or not is unaffected by the level of crime or enforcement spending. It is time to relax this assumption.

Suppose that crime takes the form of stealing from the law-abiding, and that all workers have the same chance of being robbed, and lose the same amount as a consequence. The amount of crime per non-criminal is hence $C/(n - C)$; let the amount stolen per ‘unit’ of crime be s , then each law-abiding citizen is worse off by an amount $sC/(n - C)$ because of crime. Also, we assume that taxes are paid just by the non-criminals, so the tax per worker is $E/(n - C)$. Instead of Eq. (2), the condition defining the ‘critical’ wage is now

$$pu_2 + (1 - p)u_1 = w^* - \left\{ \frac{(sC + E)}{(n - C)} \right\}, \tag{23}$$

and after some manipulation we derive the following equation for the EC locus (it is more convenient now to express it with p on the left-hand side):

$$p = \left\{ \frac{1}{(u_1 - u_2)} \right\} \left[u_1 - 2\alpha \frac{C}{m} - \underline{w} + \alpha + \frac{(sC + E)}{(n - C)} \right]. \tag{24}$$

Differentiating, we obtain an expression for the slope of the EC locus:

$$\frac{dp}{dC} \Big|_{EC} = \frac{1}{(u_1 - u_2)} \left[\frac{-2\alpha}{m} + \frac{(n + E)}{(n - C)^2} \right]. \tag{25}$$

It is clear that this may be positive, and hence the EC locus may slope upwards. The idea is that an increase in the aggregate level of crime makes becoming a criminal more attractive (for given probability of punishment), as the return to working falls, because of a rise in both crime and taxes per worker. However, each successive individual who turns to crime has a higher opportunity cost of crime; nevertheless, if the effect of aggregate crime on the (relative) attractiveness of crime is strong enough, it is possible that the first effect outweighs the second, and there hence needs to be an increase in the probability of punishment for the increase in crime to be compatible with individual rationality. In terms of Eq. (25), an upward sloping EC locus requires the second term in square brackets to be greater (in absolute value) than the first. The second term is increasing in C ; it is hence possible that the EC locus is initially downward sloping and then slopes upward; also, the EC locus is now convex.

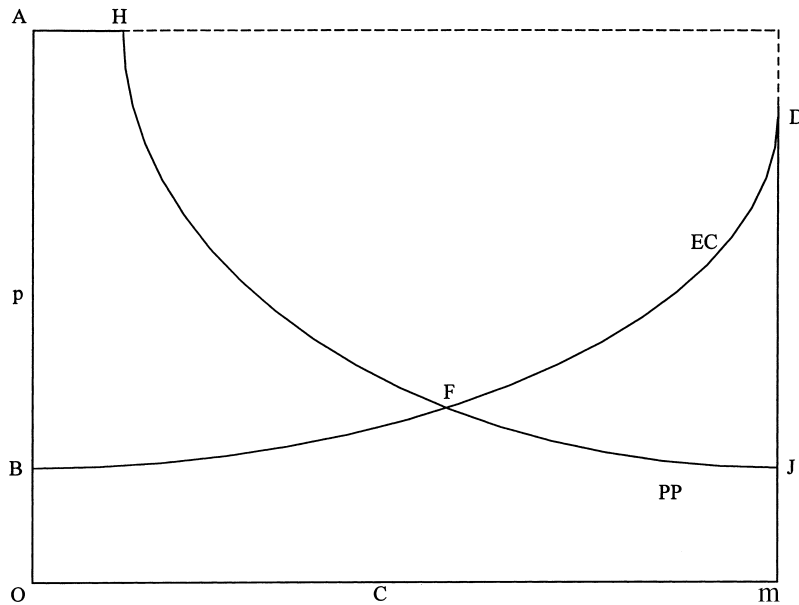


Fig. 2. An upward sloping EC locus.

An upward sloping EC locus, shown in Fig. 2, has some interesting implications. The EC locus now consists of part of the vertical axis (AB), the interior portion itself (BD) and the section of the $C = m$ locus (Dm) which lies below D. As drawn, there are three equilibria: A, F and J; given our stability criterion, the 'border' equilibria A and J will be stable whereas F will be unstable. The instability of the interior equilibrium F can be explained by the following example: suppose the system is initially at F and there is a downward movement along PP—that is, p falls and C rises. Then the attractiveness of becoming a criminal rises, both because of the fall in the probability of punishment and the general equilibrium effects of the rise in aggregate crime.

Now there can be multiple equilibria with a horizontal PP locus (for certain values of p there could be three values of C on the EC locus; so if there is a horizontal PP locus corresponding to one of these values of p , there will be three equilibria). Another implication concerns the effects of an increase in E in the high-crime equilibrium. Initially there is no effect, until the intercept of the EC locus with the $C = m$ locus is reached. When this happens (or, more precisely, when the PP locus moves just above the intercept), we are now above the EC locus. With our previous assumption about dynamics, this means that crime falls to zero, and we reach a zero-crime equilibrium. It seems then that increases in enforcement expenditure can have very different effects; sometimes they may have no effect on crime whereas in other circumstances the effects may be huge.

Many other extensions of the model can be contemplated. For example, we might suppose that individuals can take actions to reduce the probability of their being subject to crime (e.g. by installing burglar alarms). One important consideration is whether an individual who thus reduces the likelihood that he will be subject to crime reduces the overall level of crime or

merely diverts it elsewhere. Another extension is to consider the possibility that organized crime might emerge—a group of criminals might net higher returns if they combined rather than acted individually. However, such extensions would require non-trivial modifications of the model, and will not be pursued here.

6. Policy

Obviously, we would like our analysis to shed some light on policy. In order to do this, we need to represent the goals of policy makers, which we do by specifying a social welfare function. One possibility, which we adopt, is to assume that utility functions are linear in income (defined to include losses from crime), in which case the (individualistic) social welfare function is an appropriately weighted sum of agents’ net incomes. Our candidate for the social welfare function is hence

$$\underline{w}(n - m) + \left(\underline{w} + \alpha \frac{C}{m} \right) (m - C) - (sC + E) + \delta Cu, \tag{26}$$

where $u \equiv pu_2 + (1 - p)u_1$, the expected net income of a criminal.

The first term in Eq. (26) gives the wage earnings of incorruptible agents and the second term the gross earnings of employed corruptible agents; the sum of these terms gives the total gross wage bill (and also gross national product). The average wage of corruptible agents is $(w^* + \underline{w} + \alpha)/2$; using Eq. (3) this can be shown to equal $\underline{w} + \alpha C/m$, and the number of such workers is obviously $(m - C)$ – this explains the second term. The third term in Eq. (26) subtracts the total loss from crime and expenditure on enforcement, and the fourth term adds the total expected well-being of criminals, weighted by δ , which represents the weight society puts on the well-being of criminals. What is the appropriate value of δ ? We do not take a stand on this issue. Instead, we will look at the two polar cases: first, where $\delta = 1$, which means that criminals’ utility is not discounted at all by the fact that they are criminals, and is surely the maximum value it should take. The second is where $\delta = 0$, implying that criminals’ welfare does not enter into social welfare at all, and is arguably the minimum value it should take.¹³ Note that in our formulation of the social welfare function we do not weight criminals’ losses due to crime and taxes paid by criminals by δ . Although in principle we should do this, it merely complicates the analysis and does not add any essential insights (and of course it makes no difference if δ is unity). Another way of rationalizing our formulation would be to suppose that, as in the extension in Section 5, only non-criminals pay taxes and are the victims of crime. This would, however, complicate the formulation of the EC locus and mean we could not use the comparative statics results derived in Section 6. We prefer, at least initially, to combine our first, simpler, model with the social welfare analysis.

We are interested in the change in social welfare when either E or α changes; the change in social welfare can be calculated to be (we employ (3) in deriving (27)):

¹³ Supporters of ‘retributive’ punishment might argue for a value of δ less than zero. This does imply, however, that society would be better off if criminals are worse off in the absence of any change in the net incomes of non-criminals, which does seem implausible.

$$\left[\left(\frac{C}{m} \right) (m - C) \right] d\alpha + [-w^* - s + \delta u] dC - dE - \delta C(u_1 - u_2) dp. \quad (27)$$

An increase in crime reduces social welfare inasmuch as it reduces the number of workers; the marginal worker produces w^* and this is also his contribution to social welfare. However, workers who turn to crime receive a certain expected utility and it is necessary to look at the appropriate social valuation of this when evaluating social welfare. In addition, each extra criminal steals s goods and this is a net loss as far as social welfare is concerned. (The benefits to the criminal are incorporated in u_1 and u_2 .) On the assumption that $\delta = 1$, the net social cost of a small change in crime would be just $-sdC$. The explanation is that when the marginal worker becomes a criminal, he is no better off, no worse off than before, and with $\delta = 1$ this does not change social welfare (from Eq. (2), $w^* = u$ so that the term in square brackets in (27) reduces to $-s$). However, there is a net social loss because the criminal now supports himself by stealing goods, rather than producing them. If $\delta < 1$, then although the worker who switches to crime is no better off, no worse off than before, society evaluates this shift as a loss.

From Eq. (27), the optimal level of enforcement spending is given by the solution to

$$[-w^* - s + \delta u] \frac{dC}{dE} - \delta C(u_1 - u_2) \frac{dp}{dE} = 1 \quad (28)$$

Two special cases are worth noting: if $\delta = 1$, then we obtain

$$-s \frac{dC}{dE} - C(u_1 - u_2) \frac{dp}{dE} = 1. \quad (29)$$

The condition is that enforcement spending should be such that its marginal benefit equals its marginal cost, which, in our formulation, is unity. Its marginal benefit, in the case if $\delta = 1$, comprises the reduction in the goods lost through crime, less the fall in the expected welfare of criminals due to any increase in the probability of punishment.

In the special case where $\delta = 0$, we derive:

$$[-w^* - s] \frac{dC}{dE} = 1. \quad (30)$$

There are now two costs to crime; the loss due to the fact that the well-being of workers who turn to crime is now no longer included in the social welfare function and the costs due to the volume of goods stolen. Since δ is now zero criminals' welfare no longer enters the social welfare function and a change in p thus does not have any direct effect on social welfare (it only now has any effect inasmuch as it changes the level of crime). Comparing Eq. (29) and Eq. (30), it should be obvious that the optimal level of enforcement expenditure in Eq. (30) is greater.

It is, perhaps surprisingly, not possible to derive any clear-cut results of changes in income inequality (represented by changes in α) on social welfare. A lower α reduces crime, providing crime is positive but less than $m/2$ (and the other assumptions in Proposition 2 are satisfied), it also reduces total earnings of those employed, and this accounts for the ambiguity. Although a reduction in α would leave the total wage bill unchanged if everybody were employed, it raises the earnings criminals would receive were they to forsake crime (if $C < m/2$), hence, provided some crime remains, the total wage bill falls.

Finally, we consider what happens when agents vote on the level of enforcement expenditures. One complication is caused by the fact that convicted criminals usually do not vote; if we make this assumption, it follows that the number of voters is $n - P$. We adopt the median voter approach and assume that the parameters of the model are never such that the median voter is a criminal (presumably unconvicted criminals do vote). It turns out that all workers have the same preferences over the level of enforcement spending, so the crucial question is the preferred level of enforcement expenditures of a typical worker. If we assume that everybody suffers crime and pays taxes, the typical worker will want to minimise $sC + E$ which involves setting E so that $dC/dE = -1/s$. This implies a level of enforcement expenditures intermediate between that given by the solutions to Eq. (29) and Eq. (30). If it is just the workers (i.e. not the criminals) who pay taxes and suffer crime, then the appropriate minimand is $(sC + E)/(n - C)$; a certain amount of manipulation shows that the condition becomes

$$\frac{dC}{dE} = \frac{(C - m)}{(ms + E)}, \quad (31)$$

and this involves a higher level of enforcement expenditures than the previous case. This is not too surprising; raising enforcement expenditures now has the additional benefit of spreading these costs and the costs of crime over a larger number of people.

One issue worth raising is whether there is still the possibility of multiple equilibria if enforcement spending is determined endogenously. We conjecture that the possibility may still survive. Suppose we have two stable equilibria for some level of enforcement spending: one with $C=0, p=1$ and the other an interior equilibrium. Then if enforcement spending is determined endogenously, it is quite likely that the zero-crime equilibrium would continue to be chosen, with enforcement spending reduced to the minimum necessary to sustain the equilibrium. If the interior equilibrium initially obtains, then optimum enforcement may mean a shift in the equilibrium, as E changes, but the interior equilibrium continues to obtain. However, there is another possibility—that the interior equilibrium might be eliminated and the system shift to the zero-crime equilibrium. This might be done by a ‘big push’ in enforcement expenditures; there is an initial large increase which raises the PP curve above the EC locus, so the interior equilibria disappear. Crime starts falling, in accord with the dynamics postulated earlier, and we eventually reach the corner equilibrium with no crime. So, there are two possibilities (at least) if there is an interior equilibrium. The first is to adjust the level of enforcement spending until a new interior optimum is reached. The second is to spend more on enforcement than would be optimal in the first case, so as to shift the economy to a new and better equilibrium. It seems fairly clear that the second option will be more preferable, the greater the difference in social welfare levels between the two equilibria, the lower the social discount rate, the greater the speed of adjustment and the more crime responds to an increase in enforcement spending. A formal analysis would require a dynamic model which goes beyond the scope of the current analysis; however it seems clear that there would be circumstances when multiple equilibria would survive the endogenizing of policy (for example, when the discount rate is high or the speed of adjustment low.)

This Section has discussed the optimal and endogenous determination of enforcement expenditures. This is a topic of some importance, but does not seem to have been treated

in the literature (there is, of course, an extensive literature on optimal punishment, such as Shavell, 1987, which would be complementary to our analysis).

7. Conclusions

We have presented a model of crime and punishment where the decision to become a criminal is rational, in the sense that it is based on a comparison of the benefits and costs of crime with those of its alternative. A crucial assumption is that agents are heterogeneous, where the heterogeneity is specified in such a way as to allow us to derive rigorously the aggregate level of crime. An enforcement technology is also a vital component of the model, and the resultant framework enables us to consider a number of issues, both positive and normative, relating to the economic analysis of crime. We show that there can be multiple stable equilibria. We discuss the effects of changes in spending on enforcement and in inequality on crime; we also discuss policy issues. One result we derive is that (provided that crime is not too high), a reduction in inequality will reduce crime. Also, the effects of changes in enforcement spending on crime may be highly variable—there may be some circumstances under which such changes have little or no effect, other circumstances under which the effects may be large.

We believe we have presented a tractable framework, based on individual rationality and heterogeneity, for analyzing the determination of the aggregate level of crime and helping us think about many issues in the economic analysis of crime. There are many ways in which the model could be extended, some of which have already been mentioned. One interesting possibility would be to add an explicitly intertemporal dimension, which might, *inter alia*, shed some light on the possible linkages between crime and economic growth.¹⁴

Acknowledgements

I would like to thank the participants, and also Eric Bond, Richard Day, David Kelsey, Douglas Sutherland and Ping Wang, for helpful comments and suggestions. The usual disclaimer applies.

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¹⁴ Lloyd-Ellis and Marceau (1998) is a useful step in this direction.

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